A general theory of tax-smoothing

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- This paper: Build a *general* theory of optimal fiscal policy around the following *"tax-smoothing"* principle:

$$\underbrace{\text{Future taxes}}_{\text{MC of debt}} = \underbrace{\Phi \times \text{Marginal Revenue}}_{\text{MB of debt}} \tag{1}$$

- Optimality condition wrt to (some measure of) debt.
  - *LHS:* MC of issuing more debt: costly due to more taxes tomorrow.
  - *RHS*: Marginal revenue of new *debt issuance* $\times$ *social value* of relaxing the government budget.

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- *Principle:* Levy more taxes on states/dates if *MR* of debt is high
- $\Rightarrow$ *Tax more tomorrow vs today if it is cheaper to issue debt!*

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- ◂ Related literature

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  - Tax even more in bad times and *even less* in good times.
  - Random-walk results break down (no *"averaging"*).

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- Two market structures:

  1. State-contingent debt (*complete* markets) as in Lucas and Stokey (1983):

$$b_t(g^t) = \underbrace{\tau_t(g^t)w_t(g^t)h_t(g^t) - g_t}_{\text{primary surplus}} + \underbrace{\sum_{g_{t+1}} p_t(g_{t+1}, g^t)b_{t+1}(g^{t+1})}_{\text{portfolio of new debt}}$$

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  2. *Non-contingent* debt as in Aiyagari et al. (2002):

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Preferences

- *General* form of recursive utility (Kreps and Porteus (1978)):

$$V_t = u(c_t, 1 - h_t) + \beta \underbrace{H^{-1}\big(E_t H(V_{t+1})\big)}_{\text{Certainty equivalent } \mu_t}$$

- H increasing and *concave* $\Rightarrow$ aversion towards risks in V_{t+1}.
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  3. Logarithmic case, $\alpha = 1$: $H(x) = \ln x$.
- Nests the following: Epstein and Zin (1989), Weil (1990), Hansen and Sargent (2001), Tallarini (2000), Swanson (2018).

Stochastic Discount Factor

- *Two* components: Consumption (*short-run*) risk vs Continuation value risk (*long-run*):

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  - *Complete* markets: $z_t \equiv u_{ct} b_t$, *debt in MU units.*
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- Φ_t : *excess burden* (multiplier on implementability constraint) ⇒ Captures taxes.

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  - Why? $Debt_L \uparrow \Rightarrow V_L \downarrow \Rightarrow SDF_L \uparrow$: price of claims sold $\uparrow$.
  - Tax more at g_L since it becomes cheaper to issue debt against g_L.
  - Tax less at g_H because assets against g_H become more profitable ($SDF_H \downarrow$).

Excess burden with complete markets I

- Optimality condition wrt $z_{t+1} \equiv u_{c,t+1} b_{t+1}$.

Excess burden with complete markets I

- Time-additive utility: Lucas and Stokey (1983)

$$\underbrace{\Phi_{t+1}}_{\text{MC}} = \underbrace{\Phi_t}_{\text{MB}} \cdot 1, \forall t, s^t$$

- MR part trivial $\Rightarrow$ keep distortions constant over states and dates (*"tax-smoothing"*).

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- $\eta_{t+1} \equiv 0$ for time-additive utility (or for the deterministic case).

Excess burden with complete markets II

- LoM in terms of inverse excess burden of taxation

$$\frac{1}{\Phi_{t+1}} = \frac{1}{\Phi_t} - \eta_{t+1}$$

- Tax *more* tomorrow vs today ($\Phi_{t+1}(g') > \Phi_t$) when issue relatively more debt ($\eta_{t+1}(g') > 0$).

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    - Result: put *more* tax distortions on events with high u_c $\Rightarrow$ tax *even more* bad times with high u_c.

Excess burden with incomplete markets II

- Optimality condition wrt to $B_t \equiv E_t m_{t+1} u_{c,t+1} b_t$:

Excess burden with incomplete markets II

- Time-additive utility: (AMSS 2002)

$$E_t x_{t+1} \Phi_{t+1} = \Phi_t, \quad x_{t+1} \equiv \frac{u_{c,t+1}}{E_t u_{c,t+1}}$$

$\Rightarrow$ keep distortions on "average" constant (Barro (1979)).

Excess burden with incomplete markets II

- Recursive utility: LoM for the inverse *average* excess burden

$$\frac{1}{E_t n_{t+1} \Phi_{t+1}} = \frac{1}{\Phi_t} - \frac{E_{t-1} n_t \Phi_t}{\Phi_t} \cdot \xi_t \cdot b_{t-1}, \quad n_{t+1} \equiv m_{t+1} \cdot \frac{u_{c,t+1}}{E_t m_{t+1} u_{c,t+1}}$$

where ξ_t is the *relative* marginal utility adjusted by $A(x)$

$$\xi_t \equiv A(V_t) u_{ct} - A(\mu_{t-1}) E_{t-1} m_t u_{ct}$$

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- *Tax more in bad times and less in good times* $\Rightarrow$ larger weight on u_c in bad times ($q_t \uparrow$) $\Rightarrow$ amplify Aiyagari et al. (2002).
- optimal tax rate

Concluding remarks

- Minimization of welfare distortions $\Rightarrow$ tax more events against which it is *cheap* to issue debt (Taxes=$\Phi \times MR$).
- This insight holds in *all* environments $\Rightarrow$ provides a *general principle* of taxation.
- This does not mean taxes are (on average or not) *smooth*!
- Taking asset prices seriously $\Rightarrow$ *amplification* of standard taxation and debt issuance motives.

THANK YOU!

Related literature

Table: Optimal fiscal policy with time-additive utility.

	Commitment	**Discretion**
Complete Markets	Lucas and Stokey (1983) Chari et al. (1994), Zhu (1992)	Krusell et al. (2004), Occhino (2012) Debortoli and Nunes (2013)
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Table: Optimal fiscal policy with recursive utility.

	Commitment	Discretion
Complete Markets	Karantounias (2018) : EZW utility This paper: more general utility	This paper
Incomplete Markets	This paper	This paper

Value function with complete markets under commitment

- $z \equiv u_c \cdot b.$

$$V(z,g) = \max_{c \geq 0, h \in [0,1], z'_{g'} \in Z(g')} u(c, 1-h) + \beta H^{-1}\left(\sum_{g'} \pi(g'|g) H\left(V(z_{g'}, g')\right)\right)$$

subject to

$$z = \underbrace{u_c c - u_l h}_{\text{surplus}} + \underbrace{\beta \sum_{g'} \pi(g'|g) m'_{g'} z'_{g'}}_{\text{price x debt}}$$

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- Value functions in the constraint due to the SDF:

$$m'_{g'} \equiv \frac{H'\left(V(z'_{g'}, g')\right)}{H'(\mu)}, \quad \text{and} \quad \mu \equiv H^{-1}\left(\sum_{g'} \pi(g'|g) H\left(V(z'_{g'}, g')\right)\right).$$

- $m'_{g'} \equiv 1$ for *time-additive* utility.

Value function with incomplete markets under commitment

- State variable $B_t \equiv E_t m_{t+1} u_{c,t+1} \cdot b_t$.

Value function with incomplete markets under commitment

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- Commit to *average* marginal utility: promises across states g.

$$W(B_-, g_-) = \max_{c_g \geq 0, h_g \in [0,1], B_g} H^{-1}\left(\sum_g \pi(g|g_-) H\left(u(c_g, 1-h_g) + \beta W(B_g, g)\right)\right)$$

subject to

$$\frac{u_c(c_g, 1-h_g)}{\sum_g \pi(g|g_-) m_g u_c(c_g, 1-h_g)} B_- = \underbrace{u_c c_g - u_l h_g}_{\text{surplus}} + \underbrace{\beta B_g}_{\text{new debt}}, \forall g$$

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- Value functions W in m_g:

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Parametric examples and drifts

- Exponential CE:

$$\eta_{t+1} = A \cdot [z_{t+1} - E_t m_{t+1} z_{t+1}] \Rightarrow E_t m_{t+1} \eta_{t+1} = 0$$

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$$E_t \Phi_{t+1} \geq \Phi_t - Cov_t(m_{t+1}, \Phi_{t+1})$$

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$$\eta_{t+1} = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - E_t \kappa_{t+1} V_{t+1}^{-1} z_{t+1}] \Rightarrow E_t \kappa_{t+1} \eta_{t+1} = 0$$

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- Logarithmic CE:

$$\eta_{t+1} = V_{t+1}^{-1} z_{t+1} - E_t V_{t+1}^{-1} z_{t+1}$$

- $\Rightarrow 1/\Phi_t$ *martingale* wrt $\pi_t \Rightarrow$ *positive* drift wrt π_t. .

Optimal tax rate I

- Complete markets and commitment, $t \geq 1$

$$\tau_t = \frac{\Phi_t(\epsilon_{cc,t} + \epsilon_{ch,t} + \epsilon_{hh,t} + \epsilon_{hc,t})}{1 + \Phi_t(1 + \epsilon_{hh,t} + \epsilon_{hc,t})}$$

where $\epsilon_{cc} \equiv -u_{cc}c/u_c, \epsilon_{ch} \equiv u_{cl}h/u_c$ and $\epsilon_{hh} \equiv -u_{ll}h/u_l, \epsilon_{hc} \equiv u_{cl}c/u_l$, the respective *own* and *cross* elasticities.

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- Assume a utility function with *constant* elasticities

$$U(c, 1-h) = \frac{c^{1-\rho} - 1}{1-\rho} - a_h \frac{h^{1+\phi_h}}{1+\phi_h}$$

- $\Rightarrow \tau_t$ moves 1-1 with Φ_t, with law of motion

$$\frac{1}{\tau_{t+1}} = \frac{1}{\tau_t} - \frac{1}{\rho + \phi_h}\eta_{t+1}$$

Optimal tax rate II: Incomplete markets and commitment

- Power in c and h (constant Frisch): The optimal tax rate with recursive utility is

$$\tau_t = \frac{\Phi_t(\rho + \phi_h) - \rho\left[\Phi_t - E_{t-1} n_t \Phi_t\right] \frac{b_{t-1}}{c_t}}{1 - (E_{t-1} n_t \Phi_t)\xi_t b_{t-1} + \Phi_t(1 + \phi_h)}$$

- The respective tax rate for the *time-additive* case of Aiyagari et al. (2002) is

$$\tau_t = \frac{\Phi_t(\rho + \phi_h) - \rho\left[\Phi_t - \Phi_{t-1}\right] \frac{b_{t-1}}{c_t}}{1 + \Phi_t(1 + \phi_h)}$$

- if $\xi_t > 0$ (marginal utility relatively high) $\Rightarrow$ tax rate $\uparrow$.

Return

Excess burden without commitment and complete markets

- Value functions: Complete markets- MPE

Excess burden without commitment and complete markets

- Value functions: ◂ Complete markets- MPE
- Excess burden with time-additive utility:

$$\Phi_{t+1} = \Phi_t \cdot \underbrace{[1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial \mathcal{C}}{\partial B_{t+1}} \cdot B_{t+1}]}_{\propto \Phi \times MR}.$$

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- Excess burden with recursive utility:

$$\frac{1}{\Phi_{t+1}} = [1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial \mathcal{C}}{\partial b_{t+1}} \cdot b_{t+1}]^{-1} [\frac{1}{\Phi_t} - \nu_{t+1}]$$

- Relative "debt" position:

$$\nu_{t+1} \equiv A(V_{t+1}) u_{c,t+1} b_{t+1} - A(\mu_t) \cdot E_t m_{t+1} u_{c,t+1} b_{t+1}.$$

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- Relative "debt" position:

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- u'_c channel: tax more tomorrow vs today if you issue debt.
- V_{t+1}: tax more (less) if debt is relatively high (low).
- ⇒ the two incentives may *oppose* each other.

◂ Return

Excess burden without commitment and incomplete markets

- Value function: ◂ Incomplete markets -MPE
- *Excess burden* with *time-additive* utility:

$$E_t x_{t+1} \Phi_{t+1} = \Phi_t \cdot \left[1 + E_t x_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial \mathcal{C}}{\partial B_{t+1}} \cdot B_{t+1}\right]$$

where $x_{t+1} \equiv u_{c,t+1} / E_t u_{c,t+1}$

Excess burden without commitment and incomplete markets

- Value function: Incomplete markets -MPE
- *Excess burden* with *time-additive* utility:

$$E_t x_{t+1} \Phi_{t+1} = \Phi_t \cdot \left[1 + E_t x_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial \mathcal{C}}{\partial B_{t+1}} \cdot B_{t+1}\right]$$

where $x_{t+1} \equiv u_{c,t+1}/E_t u_{c,t+1}$

- *Excess burden* with recursive utility:

$$E_t n_{t+1} \Phi_{t+1} (1 - \xi_{t+1} b_t \Phi_t) = \Phi_t \left[1 + E_t n_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \mathcal{C}_{b,t+1} \cdot b_t\right]$$

- with $\xi_{t+1} \equiv A(V_{t+1}) u_{c,t+1} - A(\mu_t) E_t m_{t+1} u_{c,t+1}$.
- "Averaging" with respect to n_{t+1} measure.
- Continuation value channel depends on *relative* marginal utility ξ_{t+1}.

Return

Value function with complete markets and no commitment

- Markov-perfect equilibrium: *state variable* (b, g).

$$V(b,g) = \max_{c,h,b'_{g'}} u(c, 1-h) + \beta H^{-1}\Big(\sum_{g'} \pi(g'|g) H(V(b'_{g'}, g'))\Big)$$

subject to

$$u_c b = u_c c - u_l h + \underbrace{\beta \sum_{g'} \pi(g'|g) m'_{g'} u_c(\mathcal{C}(b'_{g'}, g'), 1 - \mathcal{H}(b'_{g'}, g')) b'_{g'}}_{\text{Recursive utility + Markov-perfect}}$$

$$c + g = h$$

- where $m'_{g'} \equiv \frac{H'(V(b'_{g'}, g'))}{H'(\mu)}$
- MPE: $c = \mathcal{C}, h = \mathcal{H}$.

Return

Value function with incomplete markets and no commitment

- State variable is *non-contingent* debt: (b_-, g).

$$V(b_-, g) = \max_{c \geq 0, h \in [0,1], b \in \mathcal{B}} u(c, 1-h) + \beta H^{-1}\Big(\sum_{g'} \pi(g'|g) H(V(b, g'))\Big)$$

subject to

$$u_c(c, 1-h)b_- = u_c c - u_l h + \beta \underbrace{\Big(\sum_{g'} \pi(g'|g) m'_{g'} u_c(\mathcal{C}(b, g'), 1 - \mathcal{H}(b, g'))\Big)}_{Average\ \text{MU}} \cdot b$$

$$c + g = h$$

where $m'_{g'} \equiv \frac{H'\left(V(b,g')\right)}{H'\left(H^{-1}\left(\sum_{g'} \pi(g'|g) H(V(b,g'))\right)\right)}$.

- MPE: $c = \mathcal{C}, h = \mathcal{H}$.

Return

Numerical exercises- Karantounias (2018)

Calibration:

- Utility function: $\rho = 1 < \gamma$

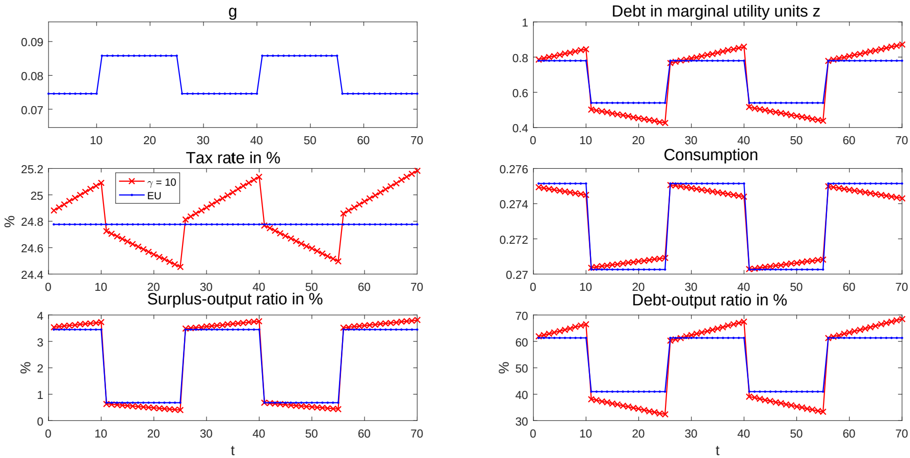
$$v_t = \ln c_t - a_h \frac{h_t^{1+\phi_h}}{1+\phi_h} + \frac{\beta}{(1-\beta)(1-\gamma)} \ln E_t \exp\big((1-\beta)(1-\gamma)v_{t+1}\big)$$

- Parameters: $(\beta, \phi_h, \gamma) = (0.96, 1, 10)$
- Shocks
  - i.i.d. shocks: mean 20% and std 2%.
  - Chari et al. (1994) shocks.

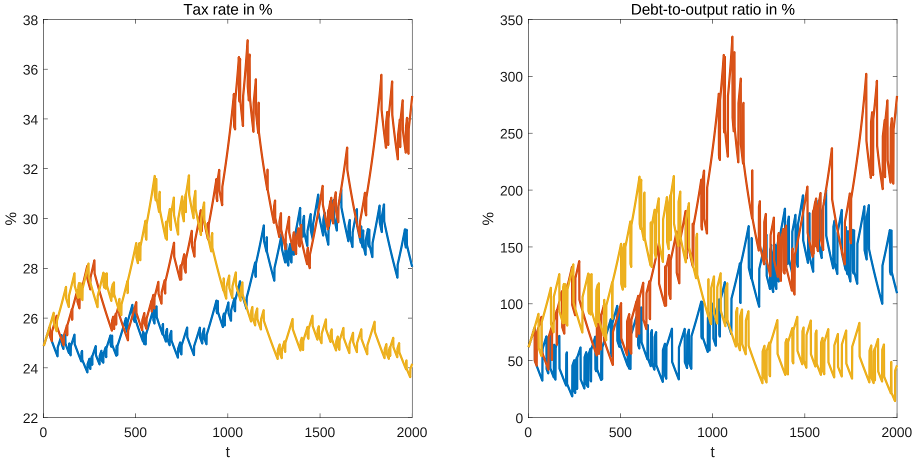
Computational issues:

- Endogenous state space.
- Lack of the contraction property due to the value function in the constraint.
- Non-convexities.

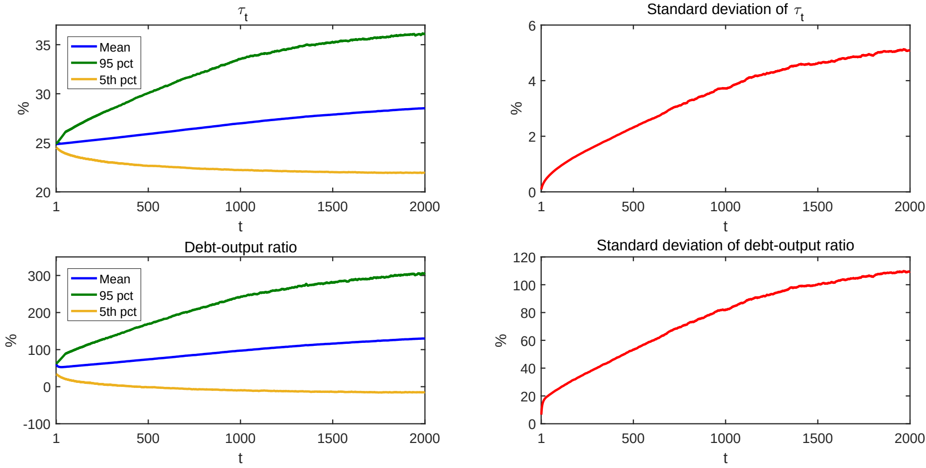
Instructive sample path



Random sample paths



Volatility and back-loading of distortions



- Positive drift.
- Increasing volatility over time, “fanning-out” of the distribution.

Stationary moments

Tax rate in %	**i.i.d.**	**CCK shocks**	**2 $\times$ std(g)**
Mean	30.86	30.49	31.26
St. Dev	4.94	5.52	7.76
St. Dev of Δ	0.17	0.41	0.90
Autocorrelation	0.9994	0.9972	0.9932

- Enormous volatility of the tax rate and therefore of debt .
- Chari et al. (1994): volatility of tax rate of 5-15 basis points.

Return

Stationary distribution: debt

debt/output in %	**i.i.d.**	**CCK shocks**	**2 $\times$ std(g)**
Mean	181.97	172.15	180.34
St. Dev	104.28	117.05	163.22
St. Dev of Δ	12.72	12.48	26.07
Autocorrelation	0.9926	0.9972	0.9877

Return

Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent, and Juha Seppala. 2002. Optimal Taxation without State-Contingent Debt. *Journal of Political Economy* 110 (6):1220–1254.

Barro, Robert J. 1979. On the Determination of the Public Debt. *Journal of Political Economy* 87 (5):940–71.

Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J. Sargent. 2017. Fiscal Policy and Debt Management with Incomplete Markets. *Quarterly Journal of Economics* 132 (1):617–663.

Chari, V.V., Lawrence J. Christiano, and Patrick J. Kehoe. 1994. Optimal Fiscal Policy in a Business Cycle Model. *Journal of Political Economy* 102 (4):617–652.

Debortoli, Davide and Ricardo Nunes. 2013. Lack of commitment and the level of debt. *Journal of the European Economic Association* 11 (5):1053–1078.

Epstein, Larry G. and Stanley E. Zin. 1989. Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57 (4):937–969.

Farhi, Emmanuel. 2010. Capital Taxation and Ownership when Markets are Incomplete. *Journal of Political Economy* 118 (5):908–948.

Hansen, Lars Peter and Thomas J. Sargent. 2001. Robust Control and Model Uncertainty. *American Economic Review* 91 (2):60–66.

Karantounias, Anastasios G. 2017. Greed versus fear: optimal time-consistent taxation with default. Federal Reserve Bank of Atlanta