

**“The constraint on public debt when $r < g$ but
 $g < m$ ”
by Ricardo Reis**

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Surrey Workshop on Macroeconomics

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 - ① Policy relevance;
 - ② Theoretical relevance.

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- The borrowing will be concentrated between mid-2021 and 2026. All borrowing will be **repaid** by 2058.
- Let $t_0 = 2021$ and $t = 2058$ then

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- Where is a perpetual deficit?

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- Where is a perpetual deficit?
- What is sustainability?



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- Where is a perpetual deficit? It could happen that in 2058, EU might decide to finance the outstanding debt by borrowing again on the market until 2088 (time \tilde{t}). But on which premises can they get favorable market rates again? That debt has to be paid!

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- What is sustainability?

$$-T_{t_0} = B_{t_0} = \frac{T_{\tilde{t}}}{(1 + i_{t_0,t})(1 + i_{t,\tilde{t}})}$$

Debt has to be paid....to get favorable market conditions, even if \tilde{t} is very far in the future.

- There is only one agent in the economy that can borrow at will and at its own chosen rates indefinitely in the future

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- ...then it becomes a paper about inflation and price determination! (Sims, 2021)

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- There is an equilibrium condition

$$\frac{Q_{t-1}}{P_t} = E_t \left\{ \sum_{T=t}^{\infty} R_{t,T} \left[\frac{T_T}{P_T} + \frac{i_t - i_t^B}{1 + i_t} \frac{B_t}{P_t} + \frac{i_t - i_t^X}{1 + i_t} \frac{X_t}{P_t} \right] \right\}$$

with

$$Q_{t-1} = (1 + i_{t-1}^X)X_{t-1} + (1 + i_{t-1}^B)B_{t-1}$$

in which i^X is interest-rate on central bank's liabilities, X , i^B is the interest-rate on treasury debt and i_t (m) is the interest-rate on illiquid securities, with $i \geq i^B \geq i^X$.

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- This is an **equilibrium** condition, not a **solvency** constraint. It just says that government's real obligation should match real revenues.



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- B is subject to a solvency condition, as discussed before, and $\{T_t\}$ has to adjust appropriately \Rightarrow the liquidity premium $i_t - i_t^B$ on government debt depends on its fiscal-backing ability.

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- If at given price P_t , equation is violated
...then P_t adjusts to square things up!

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- If the central bank implicitly backs it then treasury's debt could also not be repaid and could carry a risk-free rate, with even a convenience yield.
- ...but in this case there is no solvency requirement, and adjustments come through prices and inflation.