A general theory of tax-smoothing[*]

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Abstract

This paper organizes, reinterprets and extends the dynamic theory of optimal fiscal policy with a representative agent by using a generalized version of recursive preferences. I allow markets to be complete or incomplete and study a policymaker that acts under commitment or discretion. I highlight the underlying common principles that hide in each particular economic environment. The resulting theories are interpreted through the *excess burden of taxation*, a multiplier, whose evolution gives rise to different notions of "tax-smoothing." Variants of a law of motion in terms of the *inverse* excess burden emerge in each environment when we allow for richer asset pricing implications through recursive preferences. The basic policy prescription is simple and intuitive and revolves around interest rate manipulation: issue new debt and tax more in the future if this can lead to lower interest rates today.

Keywords: Excess burden, recursive utility, commitment, Markov-perfect, state-contingent debt, incomplete markets, martingale, fiscal hedging.

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1 Introduction

The theory of normative fiscal policy is revolving around the use of debt returns and taxes in order to maximize the utility or the representative household subject to the financing of some exogenous government expenditures. Environments can differ in terms of the type of debt that is available to the government (state-contingent or not), or in terms of the pricing of debt. Furthermore, optimal policy can be designed under different timing protocols that capture for example commitment or discretion. In all these setups, the government is always facing the tradeoff of taxing *today* versus issuing *debt* and postponing tax distortions to a future date or state. This basic tradeoff is in the heart of any optimal taxation theory.

This study emphasizes that the underlying principle behind several tax-smoothing environments is always the *same* and is captured by the optimal choice of debt. The optimal choice of debt is always governed by an optimality condition that takes schematically the following form:

$$\text{(Average) Future Taxes} = \Phi \times \text{Marginal Revenue from Debt} \tag{1}$$

The left-hand side of (1) denotes the marginal *cost* of issuing new debt. New debt is costly because it has to be repaid with distortionary taxes, so the left-hand side is always a measure of the tax burden at a future date or state. The right-hand side of (1) denotes the marginal *benefit* of issuing debt. The marginal benefit of the government depends naturally on how much additional *revenue* the government is raising. Selling debt for a particular date or state generates revenue that relaxes the government budget, leading to less taxes today. The shadow value of relaxing the government budget is captured by the multiplier $\Phi > 0$, which will be called throughout the paper the *excess burden of taxation* and will stand as an indicator of tax distortions at the second-best.

Despite its apparent simplicity, equation (1) captures some basic economics. It gives the following prescription to the policy-maker: issue more debt and tax more tomorrow, if you can achieve higher marginal revenue from debt issuance today. Therefore, independent of the particular environment, we always know that if the planner can make debt effectively *cheaper*, he should issue more debt and tax more in the future.

The crucial element in (1) is the marginal revenue part. This depends on three factors: a) the stochastic discount factor, which determines the pricing of debt and therefore the dynamic tradeoffs that the policymaker is facing; b) the market structure, i.e. the degree of state-contingency of government debt; c) the timing protocol, i.e. the policymaker's ability to commit to a plan designed at the initial period versus the other extreme of a policy-maker that maximizes every period, taking into account the fact that future policymakers will re-optimize without respecting old promises.

In the current paper I analyze all these three aspects of the marginal revenue channel in (1). To illustrate the mechanisms I build a simple economy without capital in the spirit of Lucas and Stokey (1983) with a representative household that works, pays distortionary labor taxes and invests in government securities. The government has to finance a stochastic stream of exogenous government expenditures. I use a generalized version of recursive preferences by allowing an arbitrary concave certainty equivalent of continuation utility. This allows me to nest in my analysis cases like the preferences of Epstein and Zin (1989) and Weil (1990) (EZW utility henceforth) and the risk-sensitive preferences of Hansen et al. (1999) and Tallarini (2000). Recursive preferences bring additional curvature in the pricing of future risks, leading to a higher market price of risk and to a better matching of asset pricing facts. This is crucial for designing a theory of taxation and debt that is based on realistic properties of debt returns. I extend the standard analysis of optimal fiscal policy and show the unifying perspective that (1) provides by increasing substantially the scope: I consider complete markets for government debt as in Lucas and Stokey (1983) or the case of non-state contingent debt, as in Aiyagari et al. (2002). Furthermore, I consider the case of commitment or the case of discretion, i.e. the case of a Markov-perfect policymaker that does not respect past promises and maximizes every period taking into account only the natural state variables and the fact that future policymakers will do the same. This is the notion of Markov-perfect policy in Krusell et al. (2004) and Klein et al. (2008).

The *price manipulation* that is inherent in the marginal revenue from debt issuance depends on the pricing kernel and the commitment protocol. The planner uses the increased sensitivity of the household towards future risks, and if there is lack of commitment, takes also into account that the future policymaker will not be constrained by past promises. Depending on the market structure we may have "averaging" of distortions in the LHS of (1).

Allowing for a general certainty equivalent allows me to discover that the coefficient of absolute risk aversion with respect to continuation utilities is crucial for the understanding the properties of the optimal plan. This is a generalization of Karantounias (2018), who only considered the EZW case in an environment with complete markets. Moreover, the standard results of Aiyagari et al. (2002), who treat an economy with incomplete markets and commitment and endogenize the Barro (1979) setup, are modified in a non-trivial way when we turn into recursive utility. Distortions do not stay any more on average constant. Instead, *average* distortions increase in bad times relatively to good times, amplifying therefore the under-insurance results of Aiyagari et al. (2002). The marginal cost/marginal benefit interpretation of (1) translates to a law of motion in terms of the *inverse* excess burden of taxation, that holds in some form or another, in almost all the environments I consider and for any concave certainty equivalent. And it explains powerfully the seemingly unrelated economics of the different environments.

Table 1: Optimal fiscal policy with time-additive utility.

	Commitment	**Discretion**
Complete markets	Lucas and Stokey (1983) Chari et al. (1994), Zhu (1992)	Krusell et al. (2004), Occhino (2012) Debortoli and Nunes (2013)
Incomplete markets	Aiyagari et al. (2002), Farhi (2010) Bhandari et al. (2017)	Martin (2009) Karantounias (2017)

Table 2: Optimal fiscal policy with recursive utility.

	Commitment	**Discretion**
Complete markets	Karantounias (2018) : EZW utility This paper: more general utility	This paper
Incomplete markets	This paper	This paper

1.1 Related literature

The literature is vast. Tables 1 and 2 provide a concise depiction of the state of affairs. A more detailed description will follow in the future.

2 Economy

Time is discrete and the horizon is infinite. To make my points about the debt revenue channel and the optimal determination of the excess burden of taxation over states and dates, I use an economy without capital and a representative household as in Lucas and Stokey (1983) and Aiyagari et al. (2002). Government expenditures are exogenous, stochastic, provide no utility and live in a finite set. Let g_t denote the spending shock at time t and let $g^t \equiv (g_0, g_1, ..., g_t)$ denote the partial history of shocks up to period t with probability $\pi_t(g^t)$.[1] There is no uncertainty at $t = 0$, so $\pi_0(g_0) \equiv 1$. The operator E denotes expectation with respect to π throughout the paper.

The resource constraint of the economy reads

$$c_t(g^t) + g_t = h_t(g^t), \tag{2}$$

where $c_t(g^t)$ consumption and $h_t(g^t)$ labor. The notation indicates the measurability of these functions with respect to the partial history g^t. Total endowment of time is normalized to unity,

[1] Introducing technology shocks is straightforward in this setup.

so leisure is $l_t(g^t) = 1 - h_t(g^t)$.

2.1 Preferences

The household is valuing stochastic streams of consumption and leisure using a recursive utility criterion of Kreps and Porteus (1978),

$$V_t = u(c_t, 1 - h_t) + \beta H^{-1}\big(E_t H(V_{t+1})\big), \tag{3}$$

where u an increasing and concave function of consumption and leisure, H an increasing and *concave* function, and $A(x) \equiv -H''(x)/H'(x)$ the coefficient of absolute risk aversion with respect to risks in continuation utilities. With these preferences, the household exhibits *aversion* towards volatility in continuation utilities. Current utility is the sum of period utility u and the *certainty equivalent* (CE) of continuation utility, $\mu_t \equiv H^{-1}\big(E_t H(V_{t+1})\big)$. To preserve concavity of the utility recursion, I further assume that the certainty equivalent μ_t is a concave function of future utilities, V_{t+1}.[2] Time-additive utility corresponds to risk *neutrality* with respect to risks in continuation utilities and corresponds to a linear H, $H(x) = x$, so the certainty equivalent reduces to expected future utility, $\mu_t = E_t V_{t+1}$.[3]

Consider now the stochastic discount factor (SDF) with the preferences in (3). We have

$$S_{t+1} = \beta m_{t+1} \frac{u_{c,t+1}}{u_{ct}}, \quad \text{where} \quad m_{t+1} \equiv \frac{H'(V_{t+1})}{H'(\mu_t)}. \tag{4}$$

Thus, the stochastic discount factor has an extra term m_{t+1} that represents the additional marginal utility that the household derives from continuation utility $H'(V_{t+1})$, properly scaled by the additional utility of the certainty equivalent of future utility, $H'(\mu_t)$.[4] This marginal utility is above and beyond the period marginal utility that the household derives from future consumption, $u_{c,t+1}$. We will see in the next paragraph that for some cases of the certainty equivalent, m_{t+1} will be associated with a *change of measure*. Clearly, in the time-additive or the deterministic case we have $m_{t+1} \equiv 1$.

The entire analysis throughout the paper is conducted in terms of a concave certainty equivalent in (3). Whenever I need more concrete illustrations, I will use two parametric examples for H, an

[2]A sufficient condition for that is that absolute risk tolerance, $-H'(x)/H''(x)$, is a weekly concave function. See Gollier (2004, p. 322).

[3]See Backus et al. (2004) for an expansive survey on dynamic preferences over uncertainty.

[4]For the calculation of the stochastic discount factor we need the derivative $\partial V_t/\partial c_{t+1}$. We have $\partial V_t/\partial c_{t+1} = \beta\pi_{t+1}(g_{t+1}|g^t)H^{-1\prime}(E_t H(V_{t+1}))H'(V_{t+1})\frac{\partial V_{t+1}}{\partial c_{t+1}}$ and $\frac{\partial V_t}{\partial c_t} = u_{ct}$. Recall that $H^{-1\prime}(u) = 1/H'(H^{-1}(u))$. Thus, $H^{-1\prime}(E_t H(V_{t+1})) = 1/H'(\mu_t)$, which delivers (4).

exponential and a power function.[5]

Exponential CE. Assume that H is an exponential function,

$$H(x) = -A^{-1}\exp(-Ax), A > 0. \tag{5}$$

This function delivers constant absolute risk aversion with respect to future risks, $A(x) = A$, and a certainty equivalent and m_{t+1} that are respectively

$$\mu_t = -A^{-1}\ln E_t \exp(-AV_{t+1}) \quad \text{and} \quad m_{t+1} = \frac{\exp(-AV_{t+1})}{E_t \exp(-AV_{t+1})}. \tag{6}$$

The exponential case is of particular interest because the random variable m_{t+1} can be interpreted as a conditional likelihood ratio or a change of measure, since $m_{t+1} \geq 0$ and $E_t m_{t+1} = 1$. We will refer to the induced probability measure as the *continuation-value* adjusted measure, $\pi_t \cdot M_t$, where $M_t \equiv \prod_{i=0}^t m_i, m_0 \equiv 1$.

The exponential CE is associated with various familiar cases in the literature. First, it corresponds to the risk-sensitive preferences of Hansen et al. (1999) and Tallarini (2000).[6] Furthermore, if we make the assumption that the period utility is logarithmic in the composite good of consumption and leisure, then (5) corresponds also to the Epstein and Zin (1989) and Weil (1990) (EZW henceforth) preferences with unitary elasticity of substitution.

Power CE ($\alpha \neq 1$). Assume a power function for H,

$$H(x) = \frac{x^{1-\alpha} - 1}{1-\alpha}, x > 0, \tag{7}$$

where $\alpha > 0$ and $\alpha \neq 1$. In the case of a power CE, I also make the assumption that $u > 0$, so that the utility recursion is real-valued. We obviously have $A(x) = \alpha/x$ and the respective CE and m_{t+1} take the form

$$\mu_t = \left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad m_{t+1} = \left(\frac{V_{t+1}}{\mu_t}\right)^{-\alpha} = \kappa_{t+1}^{-\frac{\alpha}{1-\alpha}}, \tag{8}$$

[5]Both of these cases furnish a concave certainty equivalent, since absolute risk tolerance is either constant or linear (see footnote 2).

[6]There is an alternative interpretation of the exponential CE in terms of the multiplier preferences of Hansen and Sargent (2001) which are designed to capture fear of model misspecification, and the household's desire for robust decision rules.

where $\kappa_{t+1} \equiv \frac{V_{t+1}^{1-\alpha}}{E_t V_{t+1}^{1-\alpha}} > 0$. Note that $E_t \kappa_{t+1} = 1$, so κ_{t+1} defines again a change of measure that applies a continuation-value adjustment, with an induced measure $\pi_t \cdot K_t$, $K_t \equiv \prod_{i=0}^{t} \kappa_i, \kappa_0 \equiv 1$. So when aversion towards future risk is expressed with a power function, m_{t+1} corresponds to a conditional likelihood ratio raised in a particular power. Besides the requirement that $u > 0$ (and the typical monotonicity and concavity assumptions for period utility), I make no further assumptions.[7] If we restrict the period utility u to also take a power form, then we have the case of EZW utility which differentiates between static risk aversion an intertemporal elasticity of substitution.[8]

Logarithmic CE, ($\alpha = 1$). Assume that $u > 0$. In the logarithmic case of $H(x) = \ln(x)$, we have $\mu_t = \exp(E_t \ln V_{t+1})$. We can define $v_t \equiv \ln V_t$ and express the utility recursion as

$$v_t = \ln[u_t + \beta \exp(E_t v_{t+1})]. \tag{9}$$

Similarly, the adjustment to the stochastic discount factor takes the form

$$m_{t+1} = \Big(\frac{V_{t+1}}{\exp(E_t \ln V_{t+1})} \Big)^{-1} = \exp\Big(-(v_{t+1} - E_t v_{t+1}) \Big), \tag{10}$$

so $E_t \ln m_{t+1} = 0$.[9]

2.2 Market structure and government policy

Complete markets. Consider now an environment with complete markets as in Lucas and Stokey (1983). There are no lump-sum taxes and the government resorts to a linear labor tax $\tau_t(g^t)$ in order to finance government expenditures. The representative household consumes, works at the pre-tax wage rate $w_t(g^t)$, and trades in a full set of Arrow securities with the government. These securities have price $p_t(g_{t+1}, g^t)$ and promise one unit of consumption if the state next period is g_{t+1} and zero otherwise. The household maximizes utility (3) subject to the budget constraint,

$$c_t(g^t) + \sum_{g_{t+1}} p_t(g_{t+1}, g^t) b_{t+1}(g^{t+1}) = (1 - \tau_t(g^t)) w_t(g^t) h_t(g^t) + b_t(g^t), \tag{11}$$

[7] The case of $u < 0$ can be treated in a similar way by using the function $H(x) = -\frac{(-x)^{1+\gamma}}{1+\gamma}, x < 0$ for $\gamma > 0$. We have $\mu_t = -\big(E_t(-V_{t+1})^{1+\gamma}\big)^{\frac{1}{1+\gamma}}$ and $m_{t+1} = \frac{(-V_{t+1})^{\gamma}}{(-\mu_t)^{\gamma}} = \kappa_{t+1}^{\frac{\gamma}{1+\gamma}}$, where $\kappa_{t+1} \equiv \frac{(-V_{t+1})^{1+\gamma}}{E_t(-V_{t+1})^{1+\gamma}} > 0$, a change of measure again since $E_t \kappa_{t+1} = 1$.

[8] See Swanson (2018) for an elaborate analysis of the effect of the labor margin on the ability to take gambles with the recursive preferences in (7).

[9] Note that m_{t+1} cannot be interpreted in that case as a conditional likelihood ratio since $E_t m_{t+1} = E_t \exp(\ln m_{t+1}) > \exp(E_t \ln m_{t+1}) = 1$, due to the convexity of the exponential function.

and the feasibility conditions on consumption and labor, $c_t \geq 0, h_t \in [0,1]$. The household is also subject to a typical no-Ponzi game condition and starts with an initial debt b_0. The government's dynamic budget constraint reads

$$b_t(g^t) = \tau_t(g^t)w_t(g^t)h_t(g^t) - g_t + \sum_{g_{t+1}} p_t(g_{t+1}, g^t)b_{t+1}(g^{t+1}) \tag{12}$$

Incomplete markets. Consider now the exactly same environment with the exception of the market structure. As in Aiyagari et al. (2002), assume that the government cannot issue state-contingent debt. Instead, it issues a risk-free discount bond, which promises one unit of consumption next period for each realization of the shock and trades at price $q_t(g^t)$. The household's budget constraint reads

$$c_t(g^t) + q_t(g^t)b_t(g^t) = (1 - \tau_t(g^t))w_t(g^t)h_t(g^t) + b_{t-1}(g^{t-1}). \tag{13}$$

Note that $b_t(g^t)$ indicates now the holdings of government debt at the beginning of period $t+1$, for each realization of the shock at $t+1$. The level of initial debt is b_{-1} and, as before, $c_t \geq 0, h_t \in [0,1]$. The household is also subject to individual borrowing constraints that will be assumed to be non-binding, so that we focus on an interior solution of the household's problem. Similarly, the government's budget constraint is

$$b_{t-1}(g^{t-1}) = \tau_t(g^t)w_t(g^t)h_t(g^t) - g_t + q_t(g^t)b_t(g^t). \tag{14}$$

2.3 Equilibrium and optimality conditions

The government policy is summarized by a stochastic process for taxes and debt $\{\tau, b\}$, where debt is either state-contingent or not.

Definition 1. *("Complete markets") A competitive equilibrium with taxes is a policy $\{\tau, b\}$, prices $\{p, w\}$ and an allocation $\{c, h, b\}$ such that a) Given $\{\tau\}$ and $\{p, w\}$, $\{c, h, b\}$ solves the household's maximization problem; b) given $\{w\}$, firms maximize profits; c) markets clear, that is, the resource constraint (2) holds.*

Definition 2. *("Incomplete markets") A competitive equilibrium with taxes is a policy $\{\tau, b\}$, prices $\{q, w\}$ and an allocation $\{c, h, b\}$ such that a) Given $\{\tau\}$ and $\{q, w\}$, $\{c, h, b\}$ solves the household's maximization problem; b) given $\{w\}$, firms maximize profits; c) markets clear, that is, the resource constraint (2) holds.*

Note that given the household's budget and the resource constraint (2) the government's budget constraint always holds, which is why we did not include it in the definition of the equilibrium.

Optimality conditions. Given the linear production function and the competitive labor markets, profit maximization implies $w_t(g^t) = 1, \forall t, g^t$. Furthermore, for both market structures, the household's labor supply condition equalizes the marginal rates of substitution between consumption and leisure to the after-tax wage,

$$\frac{u_l(g^t)}{u_c(g^t)} = 1 - \tau_t(g^t). \tag{15}$$

In the complete market case, the portfolio of Arrow securities is determined by the condition

$$p_t(g_{t+1}, g^t) = \pi_{t+1}(g_{t+1}|g^t) S_{t+1}(g^{t+1}), \tag{16}$$

where S_{t+1} the SDF in (4), which we analyzed in the previous section. With non-contingent debt we have instead

$$q_t(g^t) = \sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t) S_{t+1}(g^{t+1}). \tag{17}$$

These conditions, together with the resource constraint (2), the household's budget and the respective transversality conditions characterize fully the competitive equilibrium.

Revenue from debt issuance. As a prelude to the policy problem, consider the main objective of a benevolent planner that chooses taxes so that the utility of the representative household is maximized. What matters in this decision is the current tax rate versus the revenue that the planner can raise by issuing new debt. In the complete market setup, this revenue is captured by the market value of the government portfolio, $\sum_{g_{t+1}} p_t(g_{t+1}, g^t) b_{t+1}(g^{t+1})$. With non-contingent debt, the proper object is $q_t(g^t) b_t(g^t)$. The planner is a large player that takes into account how his decisions will affect debt revenue through both the direct effects of larger debt positions, and the indirect *pricing* effects. For both market structures, the revenue raised will depend a) on the SDF S_{t+1}, which entails continuation utilities, as stressed by Karantounias (2018) and b) on the timing protocol, that is, on assumptions on the commitment ability of the policymaker. Different commitment protocols lead to different price manipulation incentives through the SDF. For example, in the case of no commitment, the current policymaker has to take into account that the future policy maker will not be bound from past promises, a fact that will change the revenue that the current policy maker can raise by issuing debt.

3 Optimal policy with complete markets under commitment

Consider now a policymaker that chooses tax rates to maximize that utility of the representative household at $t = 0$ and commits to this policy. So the planner chooses a stochastic process for τ and b subject to the resource constraint, the budget constraints and the optimality conditions coming from the competitive equilibrium. I follow the primal approach of Lucas and Stokey (1983) and eliminate the tax rate and Arrow securities prices from (11) by using the optimality conditions (15) and (16). This allows me to express the budget constraint of the household in terms of allocations $\{c, h, b\}$ and continuation utilities $\{V_{t+1}\}$,

$$u_{ct}b_t = u_{ct}c_t - u_{lt}h_t + \beta E_t m_{t+1} u_{c,t+1} b_{t+1} \tag{18}$$

where continuation utilities V_{t+1} follow recursion (3) and affect (18) through $m_{t+1} = H'(V_{t+1})/H'(\mu_t)$, the extra term with recursive preferences that prices assets. Equation (18) denotes the dynamic *implementability* constraint, that is, the constraint that allocations have to satisfy so that they are implemented as a competitive equilibrium with taxes. In all environments I consider, the term $u_c c - u_l h$ denotes consumption net of after-tax income in marginal utility units. This term is equal in equilibrium to the government surplus in marginal utility units. Thus, we may think of (18) as the government budget (12).

Recursive formulation. I follow Kydland and Prescott (1980) and use a pseudo-state variable, whose initial value is chosen optimally, in order to capture the commitment of the planner to his past promises.[10] The environment here is a generalization of Karantounias (2018). Assume that shocks are Markov with transition density $\pi(g'|g)$. Let $z_t \equiv u_{ct}b_t$ denote debt in marginal utility units. Let the value of the commitment problem from period one onward be denoted as $V(z, g)$, when the state at $t = 1$ is (z, g). The Bellman equation takes the form

$$V(z, g) = \max_{c,h,z'_{g'}} u(c, 1-h) + \beta H^{-1}\Big(\sum_{g'} \pi(g'|g) H\big(V(z_{g'}, g')\big)\Big)$$

subject to

[10] The initial period problem that determines the optimal value of the state variable both in the complete and the incomplete markets case is stated in the Appendix.

$$z = u_c(c, 1-h)c - u_l(c, 1-h)h + \beta \sum_{g'} \pi(g'|g) m'_{g'} z'_{g'} \tag{19}$$

$$c + g = h \tag{20}$$

$$c \geq 0, h \in [0, 1], z'_{g'} \in Z(g') \tag{21}$$

where

$$m'_{g'} = \frac{H'(V(z'_{g'}, g'))}{H'(\mu)}, \quad \text{and} \quad \mu = H^{-1}\Big(\sum_{g'} \pi(g'|g) H\big(V(z'_{g'}, g')\big)\Big).$$

Thus, the planner is effectively choosing taxes and state-contingent debt subject to government budget constraint (19) and the resource constraint (20).

Analysis. Let $\Phi \geq 0$ denote the multiplier on the implementability constraint (19). I will call it the *excess burden of taxation* and will be the main object of interest throughout the different market environments and policy protocols that I consider. Note that $\Phi = 0$ when lump-sum taxes are available. Let also $R(\{z'_{g'}\}_{g'}) \equiv \sum_{g'} \pi(g'|g) m'_{g'} z'_{g'}$ denote the revenue from debt issuance, in marginal utility units. Then the optimality condition with respect to debt takes the form

$$\underbrace{-V_z(z_{g'}, g')}_{\text{MC: } \Phi'_{g'}} = \Phi \cdot \underbrace{[1 - V_z(z'_{g'}, g') \eta'_g]}_{\frac{\partial R}{\partial z'_{g'}}}, \tag{22}$$

where

$$\eta'_{g'} \equiv A\big(V(z'_{g'}, g')\big) z'_{g'} - A(\mu) \sum_{g'} \pi(g'|g) m'_{g'} z'_{g'}. \tag{23}$$

The optimality condition with respect to debt takes the marginal benefit/ marginal cost form (1) that I stressed in the introduction. The left-hand denotes the welfare cost of new debt, since it has to be repaid with distortionary taxes. Note from the envelope condition that $V_z(z, g) = -\Phi < 0$, so the left-hand side is equal to the future excess burden of taxation at g', $\Phi'_{g'}$. The right-hand side denotes the shadow value of the marginal revenue that the planner is raising. By issuing more debt, the planner is raising more revenue, but he has also to see how additional debt affects equilibrium prices through continuation values. An increase in debt reduces continuation values, which increases the price of an Arrow security at g' due to the aversion to utility volatility. However, prices are interconnected through the certainty equivalent in (4), so the planner has to take into account how the change in z'_g affects the pricing of the rest of the portfolio of state-contingent debt. The total pricing effect of continuation values is captured by the variable $\eta'_{g'}$, which denotes the *relative debt position* in marginal utility units of the government, adjusted properly by coefficient of *absolute risk aversion* at $V'_{g'}$, $A(V'_{g'})$, relative to the total value of the portfolio of Arrow claims, adjusted by absolute risk aversion at the certainty equivalent, $A(\mu)$. We can collect terms and rewrite (22) in terms of the *inverse* of the excess burden of taxation. The following proposition summarizes the results.

Proposition 1. *("Excess burden and optimal tax with complete markets under commitment")*

- *Turn into sequence notation and assume that $\Phi_t > 0$. The inverse of the excess burden follows the law of motion*

$$\frac{1}{\Phi_{t+1}} = \frac{1}{\Phi_t} - \eta_{t+1}, t \geq 0 \tag{24}$$

  where $\eta_{t+1} \equiv A(V_{t+1})z_{t+1} - A(\mu_t)E_t m_{t+1} z_{t+1}$, the relative debt position in marginal utility units.

  - *Assume the exponential CE (5) with m_{t+1} given in (6). Then, $\eta_{t+1} = A \cdot [z_{t+1} - E_t m_{t+1} z_{t+1}]$, so $1/\Phi_t$ is a martingale with respect to the continuation-value adjusted measure $\pi_t \cdot M_t$, since $E_t m_{t+1}\eta_{t+1} = 0$.*
  - *Assume the power CE (7), $\alpha \neq 1$ with m_{t+1} given in (8). Then, $\eta_{t+1} = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - E_t \kappa_{t+1} V_{t+1}^{-1} z_{t+1}]$, so $1/\Phi_t$ is a martingale with respect to $\pi_t \cdot K_t$, since $E_t \kappa_{t+1}\eta_{t+1} = 0$.*
  - *Assume a logarithmic CE, $\alpha = 1$ with m_{t+1} given in (10). Then, $\eta_{t+1} = V_{t+1}^{-1} z_{t+1} - E_t V_{t+1}^{-1} z_{t+1}$, so $1/\Phi_t$ is a martingale with respect to the physical measure π, since $E_t \eta_{t+1} = 0$.*

- *The optimal tax rate for $t \geq 1$ takes the form*

$$\tau_t = \frac{\Phi_t(\epsilon_{cc,t} + \epsilon_{ch,t} + \epsilon_{hh,t} + \epsilon_{hc,t})}{1 + \Phi_t(1 + \epsilon_{hh,t} + \epsilon_{hc,t})}, \tag{25}$$

  where $\epsilon_{cc} \equiv -u_{cc}c/u_c, \epsilon_{ch} \equiv u_{cl}h/u_c$ and $\epsilon_{hh} \equiv -u_{ll}h/u_l, \epsilon_{hc} \equiv u_{cl}c/u_l$, the respective own and cross elasticities of the period marginal utility of consumption and the marginal disutility of labor.

Proof. To derive (24), collect terms that involve the derivative of the value function V_z in (22), and use the envelope condition $V_z(z, g) = -\Phi$ to get $\Phi_{t+1} = \frac{\Phi_t}{1-\eta_{t+1}\Phi_t}$. When $\Phi_t = 0$, the excess burden remains at zero for every state or date afterwards. Otherwise, we can invert and get the desired law of motion. The derivation of the relative debt position for the power case deserves a comment. Note that $\eta_{t+1} = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - \mu_t^{-1} E_t \left(\frac{V_{t+1}}{\mu_t}\right)^{-\alpha} z_{t+1}] = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - E_t \left(\frac{V_{t+1}}{\mu_t}\right)^{1-\alpha} V_{t+1}^{-1} z_{t+1}]$, which delivers the expression in the proposition since $\kappa_{t+1} = V_{t+1}^{1-\alpha}/\mu_t^{1-\alpha}$. The derivation of the optimal tax rate in (25) uses the same steps as in Karantounias (2018) and is provided in the Appendix for completeness. □

Comments. The law of motion (24) provides a generalization of the analysis in Karantounias (2018) for a general certainty equivalent. It uncovers the crucial role that absolute risk aversion $A(x)$ plays in the manipulation of state-contingent prices, which is a novel feature of the analysis. Furthermore, (24) shows that the law of motion in terms of the *inverse* excess burden of taxation is a fundamental feature of the analysis, that depends on neither the period utility nor the particular parametric specification of the certainty equivalent. This feature of the analysis was not clear with EZW utility.

The increment η_{t+1} in the law of motion (24) and the respective dynamics of the excess burden depends obviously on the certainty equivalent specification. Note that Φ_t is a submartingale with respect to the proper measures for the three examples that we considered, so we have the presence of positive drifts, generalizing again Karantounias (2018). We typically expect that $\eta_{t+1} > 0$ for good shocks (the government issues against them relatively high debt) and $\eta_{t+1} < 0$ for bad shocks (high g is hedged with relatively lower positions) , leading therefore to higher taxes in good times and lower taxes in bad times.[11]

4 Optimal policy with incomplete markets under commitment

Consider now our second market structure which features non-contingent debt as in Aiyagari et al. (2002). Price manipulation in that case means that the planner is trying to affect *"average"* marginal utilities, as we can see from the Euler equation (17). So the richer SDF will force the planner to consider how continuation values affect the average price of debt over states. And the respective marginal revenue from debt issuance will be contrasted to the *average* tax distortions next period.

[11] See Karantounias (2018) for a thorough quantitative analysis.

4.1 Preliminaries

To set up the commitment problem with incomplete markets, we need to have a planner who realizes how his decisions affects his promises *across* states. For that reason, we express the utility recursion (3) in terms of the certainty equivalent, *before* the realization of the shock at time t:

$$\mu_{t-1} = H^{-1}\Big(E_{t-1}H\big(u(c_t, 1-h_t) + \beta\mu_t\big)\Big). \tag{26}$$

Following the primal approach, use the labor supply condition (15) and the Euler equation (17) to eliminate tax rates and interest rates from the household's budget constraint (13). This leads to the following implementability constraint with incomplete markets,

$$u_{ct}b_{t-1} = u_{ct}c_t - u_{lt}h_t + \beta E_t m_{t+1}u_{c,t+1}b_t. \tag{27}$$

Define $B_t \equiv E_t m_{t+1}u_{c,t+1}b_t$, which is now a measure of debt in *average marginal utility* units. This variable captures the planner's past promises of period marginal utility and continuation values across states and serves as a state variable in a recursive formulation of the problem. The implementability constraint (27) becomes then

$$\frac{u_{ct}}{E_{t-1}m_t u_{ct}}B_{t-1} = u_{ct}c_t - u_{lt}h_t + \beta B_t. \tag{28}$$

I also assume that the planner is subject to some upper (lower) debt (asset) limits, which can be stricter than the natural borrowing limits. I follow Farhi (2010) and express the debt limits directly in terms of the variable B_t, so $\underline{B}_t \leq B_t \leq \bar{B}_t$.

For future reference, define the random variable $x_t \equiv \frac{u_{ct}}{E_{t-1}m_t u_{ct}}$. In a world with time-additive utility, this variable will capture the *risk-adjusted* measure, since it adjusts for the household's aversion to consumption volatility. In a world with recursive utility, we can define one more random variable, $n_t \equiv m_t \cdot x_t \geq 0$. We obviously have $E_{t-1}n_t = 1$, so n_t is a change of measure with respect to the physical measure π, and it entails an adjustment for *consumption* risk (through x_t) and an adjustment for *continuation-value* risk through m_t.[12] If we followed the terminology of Bansal and Yaron (2004) and Hansen et al. (2008), these adjustments would correspond to short- (x_t) and long-run risk (m_t). The induced measure $\pi_t \cdot N_t$, $N_t \equiv \prod_i^t n_i$, is instrumental in our analysis of tax smoothing with incomplete markets.[13]

[12] Note that we obviously have $n_t = S_t/E_{t-1}S_t$, where S_t the stochastic discount factor in (4).

[13] When m_t can be interpreted as a change of measure with respect to π, (as in the exponential case (5)), then $E^m_{t-1}x_t = 1$, where E^m refers to the expectation with respect to m. In that case, x_t would be a change of measure with respect to the induced continuation-value adjusted measure $\pi_t \cdot M_t$, so n_t would be a product of two conditional

4.2 Recursive formulation

Let now the shocks be Markov. Let $W(B_-, g_-)$ denote the optimal value of the certainty equivalent, when the state is (B_-, g_-), where the underscore "_" denotes previous period. The Bellman equation takes the following form:

$$W(B_-, g_-) = \max_{c_g, h_g, B_g} H^{-1}\Big(\sum_g \pi(g|g_-) H\big(u(c_g, 1-h_g) + \beta W(B_g, g)\big)\Big)$$

subject to

$$\frac{u_c(c_g, 1-h_g)}{\sum_g \pi(g|g_-) m_g u_c(c_g, 1-h_g)} B_- = u_c(c, 1-h_g)c_g - u_l(c_g, 1-h_g)h_g + \beta B_g, \forall g \tag{29}$$

$$c_g + g = h_g, \forall g \tag{30}$$

$$c_g \geq 0, h_g \in [0,1], \underline{B}_g \leq B_g \leq \bar{B}_g, \forall g, \tag{31}$$

where m_g is shorthand for the continuation-value adjustment in the SDF,

$$m_g = \frac{H'\big(u(c_g, 1-h_g) + \beta W(B_g, g)\big)}{H'\Big(H^{-1}\big(\sum_g \pi(g|g_-) H(u(c_g, 1-h_g) + \beta W(B_g, g))\big)\Big)}, \forall g. \tag{32}$$

Given the timing of the problem, the planner is choosing now state-contingent consumption, labor and "debt", (c_g, h_g, B_g). This is the reason why we have an implementability and resource constraint for each realization of the shock g. Note also that in the case of independent shocks, we would not need to keep track of g_-. As in the complete markets case, value functions show up in the constraints through the determination of the price of risk-free debt.

4.3 Analysis

Consider first the optimality condition with respect to B_g, which will determine the optimal second-best distortions over states and dates. In the Appendix I show that this takes the form

$$\underbrace{-W_B(B_g, g)}_{\text{MC: average future excess burden}} = \Phi_g + \underbrace{\Big(\sum_g \pi(g|g_-) n_g \Phi_g\Big)}_{\text{value of relaxing IC across } g} \cdot \underbrace{\big(-W_B(B_g, g)\big) \cdot \xi_g b_-}_{\text{change in revenue due to } \partial W}, \tag{33}$$

where, recalling the definition of the state variable B, b_- stands for non-contingent debt in the

likelihood ratios.

beginning of the period, $b_- = B_- / \sum_g \pi(g|g_-) m_g u_c(c_g, 1-h_g)$,$\Phi_g$ stands for the (scaled) multiplier on the implementability constraint (29), n_g stands for the consumption and continuation-value adjusted measure, $n_g = m_g \cdot x_g$, and

$$\xi_g \equiv A(V_g) u_c(c_g, 1-h_g) - A(\mu_-) \sum_g \pi(g|g_-) m_g u_c(c_g, 1-h_g), \tag{34}$$

the *relative marginal utility* position at g, adjusted properly by absolute risk aversion at V_g and at the respective certainty equivalent at the beginning of the period, μ_-.[14]

Equation (33) takes the same form as (1). The left-hand side denotes again the cost of issuing debt that has to be repaid with distortionary taxes. From the envelope condition we have $W_B(B_-, g_-) = -\sum_g \pi(g|g_-) n_g \Phi_g$, so the left-hand side denotes future *average* distortions, since in contrast to (22), debt is now non-contingent. The right-hand side of (33) has two parts: the first part denotes the relaxation of the government budget constraint at g which bears shadow benefit Φ_g. This is coming from new debt issuance, without taking into account any price-manipulation through continuation values. But increasing debt reduces utility and increases therefore prices($-W_B > 0$), since it affects "average" marginal utility. The benefit or cost of this action in terms of revenue will depend on the *relative* marginal utility (adjusted by absolute risk aversion), ξ_g, times the amount of non-contingent debt b_-. Note that since debt is not state-contingent and since the planner operates under commitment, his marginal utility and continuation value promises are bound by the average value of this promises he is committing to. Thus, any price change through B_g affects the implementability contraints at $\tilde{g} \neq g$. This is why the price effect of continuation values in (33) is multiplied by the shadow valuation of relaxing the budget constraints *across* shocks g, $\sum_g \pi(g|g_-) n_g \Phi_g$.

After providing this interpretation, we can use sequence notation and finally derive an inverse law of motion for the excess burden of taxation.

Proposition 2. *("Excess burden of taxation with incomplete markets and commitment")*

- *The law of motion for the excess burden of taxation for $t \geq 0$ is*

$$\frac{1}{E_t n_{t+1} \Phi_{t+1}} = \frac{1}{\Phi_t} - \frac{E_{t-1} n_t \Phi_t}{\Phi_t} \cdot \xi_t b_{t-1}, \tag{35}$$

where $\xi_t \equiv A(V_t) u_{ct} - A(\mu_{t-1}) E_{t-1} m_t u_{ct}$. We have $\xi_0 \equiv 0$ so $E_0 n_1 \Phi_1 = \Phi_0$.

[14] V_g is shorthand for $V_g = u(c_g, 1-h_g) + \beta W(B_g, g)$.

- *("Drifts") Assume the planner issues debt, $b_{t-1} > 0$. Then if $\xi_t > 0$, the average excess burden increases, $E_t n_{t+1}\Phi_{t+1} > \Phi_t$. If $\xi_t < 0$, then the average excess burden decreases, $E_t n_{t+1}\Phi_{t+1} < \Phi_t$.*

- *Examples:*

  - *Assume the exponential CE in (5). Then $\xi_t = A[u_{ct} - E_{t-1}m_t u_{ct}]$, so ξ_t takes positive and negative values with $E_{t-1}m_t\xi_t = 0$. If also period utility is <u>linear</u> in consumption, then $\xi_t \equiv 0, \forall t$, $n_t = m_t$, and $E_t m_{t+1}\Phi_{t+1} = \Phi_t$.*
  - *Assume the power CE in (7). Then $\xi_t = \alpha[V_t^{-1}u_{ct} - E_{t-1}\kappa_t V_t^{-1}u_{ct}]$, so ξ_t takes positive and negative values with $E_{t-1}\kappa_t\xi_t = 0$.*
  - *Assume the logarithmic CE. Then $\xi_t = V_t^{-1}u_{ct} - E_{t-1}V_t^{-1}u_{ct}$, so ξ_t takes positive and negative values with $E_{t-1}\xi_t = 0$.*

Proposition 2 furnishes a law of motion that involves the inverse *average* excess burden of taxation. Note that the tax-smoothing results of Aiyagari et al. (2002), who provide the foundation of the Barro (1979) analysis, are nested in (35). To see that, note that with time-additive utility we have $m_t \equiv 1$, $\xi_t \equiv 0$ and $n_t = x_t = u_{ct}/E_{t-1}u_{ct}$. Then (35) becomes $E_t x_{t+1}\Phi_{t+1} = \Phi_t$, which is the martingale result of Aiyagari et al. (2002). This result is still in the general form of (1), since it equates "average" future tax distortions with the benefit of relaxing the current government budget. The marginal revenue part of the analysis is elementary, since with time-additive utility and commitment the price manipulation we have been highlighting is absent. Note furthermore the similarity of the law of motion in (35) to the respective law of motion with complete markets in (24), taking into account the proper modifications for market incompleteness. Instead of the future excess burden in (24) we have the average excess burden in (35). With market completeness the term $E_{t-1}n_t\Phi_t/\Phi_t$ in (35) would drop since the planner is not bound to keep track of his promises across states. And the relative debt position in marginal utility units (adjusted by absolute risk aversion) η_t in (24), becomes naturally with incomplete markets the relative marginal utility (adjusted by absolute risk aversion) ξ_t *times* non-state contingent debt, b_{t-1}.

Proposition 3. *("Optimal tax rate with incomplete markets under commitment") The optimal tax rate for $t \geq 1$ takes the form*

$$\tau_t = \frac{\Phi_t(\epsilon_{cc,t} + \epsilon_{ch,t} + \epsilon_{hh,t} + \epsilon_{hc,t}) - (\epsilon_{cc,t} + \epsilon_{hc,t})\left[\Phi_t - E_{t-1}n_t\Phi_t\right]\frac{b_{t-1}}{c_t}}{1 - (E_{t-1}n_t\Phi_t)\xi_t b_{t-1} + \Phi_t(1 + \epsilon_{hh,t} + \epsilon_{hc,t}) - \epsilon_{hc,t}\left[\Phi_t - E_{t-1}n_t\Phi_t\right]\frac{b_{t-1}}{c_t}} \tag{36}$$

By using the law of motion (35), (36) can be rewritten as function of the "average" excess burden of taxation,

$$\tau_t = \frac{E_t n_{t+1}\Phi_{t+1}(\epsilon_{cc,t}+\epsilon_{ch,t}+\epsilon_{hh,t}+\epsilon_{hc,t}) - (\epsilon_{cc,t}+\epsilon_{hc,t})\left[E_t n_{t+1}\Phi_{t+1} - \frac{1}{1/(E_{t-1}n_t\Phi_t)-\xi_t b_{t-1}}\right]\frac{b_{t-1}}{c_t}}{1 + E_t n_{t+1}\Phi_{t+1}(1+\epsilon_{hh,t}+\epsilon_{hc,t}) - \epsilon_{hc,t}\left[E_t n_{t+1}\Phi_{t+1} - \frac{1}{1/(E_{t-1}n_t\Phi_t)-\xi_t b_{t-1}}\right]\frac{b_{t-1}}{c_t}} \quad (37)$$

Proof. See the Appendix for the calculations behind propositions 2 and 3. To get (37) I divided each term of (36) over $1-(E_{t-1}n_t\Phi_t)\xi_t b_{t-1}$ and used that fact that $E_t n_{t+1}\Phi_{t+1} = \Phi_t/(1-(E_{t-1}n_t\Phi_t)\xi_t b_{t-1})$ from (35). □

Tax rate in Aiyagari et al. (2002). Note that in the time-additive case of Aiyagari et al. (2002) the optimal tax rate (36) simplifies to

$$\tau_t = \frac{\Phi_t(\epsilon_{cc,t}+\epsilon_{ch,t}+\epsilon_{hh,t}+\epsilon_{hc,t}) - (\epsilon_{cc,t}+\epsilon_{hc,t})\left[\Phi_t - \Phi_{t-1}\right]\frac{b_{t-1}}{c_t}}{1+\Phi_t(1+\epsilon_{hh,t}+\epsilon_{hc,t}) - \epsilon_{hc,t}\left[\Phi_t-\Phi_{t-1}\right]\frac{b_{t-1}}{c_t}}, \quad (38)$$

where I used the fact that $E_{t-1}x_t\Phi_t = \Phi_{t-1}$. Note that this formula reflects two channels: the time-varying excess burden of taxation (due to market incompleteness) and the costs and benefits of manipulation of legacy debt through marginal utility under commitment (this is the term that entails the difference $\Phi_t - \Phi_{t-1}$). Note that if we had state-contingent debt, this term would be absent (even with recursive utility). It will also be modified in the case without commitment (since there is no keeping of past promises).

Amplification. The law of motion (35) shows that when the planner issues debt, $b_{t-1} > 0$, then the relative marginal utility position is crucial for the allocation of tax distortions. For instance, if marginal utility is relatively high, in the sense that $A(V_t)u_{ct} > A(\mu_{t-1})E_{t-1}m_t u_{ct}$ so that $\xi_t > 0$, then average tax distortions *increase*, $E_t n_{t+1}\Phi_{t+1} > \Phi_t$. If we assume *constant* absolute risk aversion, $A(x) = A$, this statement depends only on marginal utility. We typically expect that marginal utility is high when government expenditures are high since they reduce consumption. So the planner uses the continuation value channel to *amplify* average tax distortions in bad states, and does the opposite in good states. This is an amplification of the mechanisms in Aiyagari et al. (2002). Note furthermore that this is in contrast to the analysis of proposition 1 and the results of Karantounias (2018). The relative debt position in marginal utility units η_{t+1} matters with complete markets. The sign of this object is typically dominated from the actual debt position of the government which varies across good and bad shocks when we have state-contingent debt.[15] The fiscal hedging of the government commands issuing debt against good states (low g) and have less debt (more assets) fot he bad states with high g. This leads to having a positive η_{t+1} in good times and a negative in bad times, and thus high tax rates in good times versus bad times,

[15]See the numerical analysis in Karantounias (2018).

amplifying essentially the mechanisms of Lucas and Stokey (1983). Effectively, the difference is coming from manipulating the prices of state-contingent debt adjusted by marginal utility, versus the manipulation of average marginal utility that takes place with incomplete markets. It is obvious that none of these marginal revenue effects takes place when we are in the time-additive case.

5 The case of no commitment

Consider now the same problem where before, when the government cannot commit to its promises. I am assuming a Markov-perfect planner that keeps track only of the payoff-relevant state variables of debt and the exogenous shock, (b, g). The new element that enters the problem is the fact that the planner is taking into account that the future policymaker will choose optimally from next period's perspective. This has two implications: a) the planner will try to devalue current debt ("legacy" debt), which happens only in the first-period of the commitment problem and is the source of the time-inconsistency in this setup b) the planner is going to take into account how his actions affect the actions of the future policymaker. In particular, the planner takes into account that issuing more debt for the future is going to make the future policy maker tax more, and therefore reduce consumption and interest rates, by an increase of marginal utility. Thus, given the lack of commitment, there is an incentive to issue more debt for tomorrow, i.e postpone taxation, because government debt becomes cheaper to issue.

5.1 A quick digression: time-additive utility

In order to make the mechanism clear, consider an environment with time-additive utility and complete markets. This would be a version of Krusell et al. (2004) with shocks and complete markets, or Debortoli and Nunes (2013) and Occhino (2012) with stochastic exogenous government spending shocks.

Let $\mathcal{C}, \mathcal{H}$ be the policy functions of the planner next period, as functions of the state (b, g). The problem of the time-consistent planner takes the following form:

$$V(b, s) = \max_{c,h,b'_{g'}} u(c, 1-h) + \beta \sum_{g'} \pi(g'|g) V(b'_{g'}, g')$$

subject to

$$u_c(c,1-h)b = u_c(c,1-h)c - u_l(c,1-h)h + \beta \sum_{g'} \pi(g'|g) u_c(\mathcal{C}(b'_{g'},g'), 1-\mathcal{H}(b'_{g'},g'))b'_{g'}$$
$$c + g = h$$

At the MPE we require that $c = \mathcal{C}, h = \mathcal{H}$. Assign again the multipliers Φ and λ on the implementability constraint and the resource constraint respectively. We have the following first-order necessary conditions for an interior solution, where we defined $\Omega(c,h) \equiv u_c(c,1-h)c - u_l(c,1-h)h$ and $\Omega_i, i = c, h$ the respective partial derivatives.

$$\begin{aligned}
c: &\quad u_c + \Phi[\Omega_c - u_{cc}b] = \lambda \\
h: &\quad u_l - \Phi[\Omega_h + u_{cl}b] = \lambda \\
b'_{g'}: &\quad \pi(g'|g)V_b(b'_{g'},g') + \Phi\frac{\partial \omega}{\partial b'_{g'}} = 0
\end{aligned}$$

The optimality conditions with respect to c, h take now into account the devaluing of *inherited* debt.

Note that the optimality condition with respect to *new* debt has as usual the same interpretation as (1). Consider now the determination of marginal revenue in an environment without commitment.

$$\frac{\partial \omega}{\partial b'_{g'}} = \pi(g'|g)\left[u'_c + \underbrace{(u'_{cc} - u'_{cl})\mathcal{C}_b b'_{g'}}_{\text{MU pricing effect}}\right]$$

I have used the fact that $\mathcal{C}_b = \mathcal{H}_b$. Primes $'$ denote next period. The first term in the marginal revenue is only the increase that is coming from a higher position, given prices. This is the only effect with time-additive utility and commitment. But a Markov-perfect planner will take into account that the future planner will reduce consumption in the future and increase tax rates in order to pay back the issued debt. This will increase marginal utility and increase therefore the price of debt sold and thus, the revenue to the government. This effects are in $u'_{cc}\mathcal{C}_b$, where it is assumed that $\mathcal{C}_b < 0$ (this is so in a smooth equilibrium with exogenous g). Obviously if $b'_{g'} > 0$, that is, if the planner issues debt against g', then the pricing effect of marginal utility is positive.

Consider again the first-order condition with respect to debt. This becomes

$$-V_b(b_{g'},g') = \Phi\left[u'_c + (u'_{cc} - u'_{cl})\mathcal{C}_b b'_{g'}\right]$$

Use now the envelope condition $V_b = -\Phi u_c$ and simplify to get

$$\Phi'_{g'} = \Phi\big[1 + \frac{u'_{cc} - u'_{cl}}{u'_c}\mathcal{C}_b b'_{g'}\big]$$

So, the government increases distortions for the future because it is cheaper to issue debt for next period/state. Note that as long as $b'_{g'} > 0$, and $C_b \leq 0$, we have $\Phi'_{g'} \geq \Phi, \forall g$. In sequential notation, the law of motion of the excess burden becomes

$$\Phi_{t+1} = \Phi_t\big[1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}}\mathcal{C}_b(b_{t+1}, g_{t+1})b_{t+1}\big]$$

5.2 Recursive utility and complete markets

Consider now the same problem with recursive utility. In addition to the previous analysis, the planner will manipulate equilibrium prices through the continuation value channel.

The Bellman equation takes the form

$$V(b,g) = \max_{c,h,b'_{g'}} u(c, 1-h) + \beta H^{-1}\big(\sum_{g'} \pi(g'|g) H(V(b'_{g'}, g'))\big)$$

subject to

$$u_c(c,1-h)b = u_c(c,1-h)c - u_l(c,1-h)h + \beta\sum_{g'} \pi(g'|g) m'_{g'} u_c(\mathcal{C}(b'_{g'}, g'), 1 - \mathcal{H}(b'_{g'}, g'))b'_{g'}$$

$$c + g = h$$

where

$$m'_{g'} \equiv \frac{H'(V(b'_{g'}, g'))}{H'(\mu)}, \quad \mu \equiv H^{-1}\big(\sum_{g'} \pi(g'|g) H(V(b'_{g'}, g'))\big)$$

Assign again multipliers Φ and λ respectively. The first-order conditions with respect to (c,h) are the same as in the time-additive case. The first-order condition with respect to $b'_{g'}$ becomes

$$-\pi(g'|g)m'_{g'}V_b(b'_{g'},g') = \Phi\frac{\partial\omega}{\partial b'_{g'}},$$

taking again the familiar (by now) form of (1). The marginal revenue now will reflect *two* channels: the continuation value channel (that is present also under commitment but is not present in the time-additive case) and the marginal utility channel (that is present in the standard case when a planner cannot commit):

$$\frac{\partial\omega}{\partial b'_{g'}} = \pi(g'|g)m'_{g'}\big[V_b(b_{g'},g')\nu'_{g'} + (u'_{cc}-u'_{cl})\mathcal{C}_b b'_{g'} + u'_c\big]$$

where

$$\nu'_{g'} \equiv -\big[A(V(b'_{g'},g'))u'_c b'_{g'} - A(\mu)\omega\big]$$

The marginal revenue from state-contingent debt captures the channel I mentioned before. Note that the relative debt position depends again on the value of debt in marginal utility units scaled by the coefficient of absolute risk aversion, but takes into account that policy will be optimal from next period onward. Going to the optimal debt issuance equation, and using again the envelope condition $V_b = -\Phi u_c$, we finally get

$$\Phi'_{g'} = \frac{\Phi}{1+\Phi\nu'_{g'}}\big[1+\frac{u'_{cc}-u'_{cl}}{u'_c}\mathcal{C}_b b'_{g'}\big]$$

Taking inverses, this law of motion can also be written as

$$\frac{1}{\Phi'_{g'}} = \big[1+\frac{u'_{cc}-u'_{cl}}{u'_c}\mathcal{C}_b b'_{g'}\big]^{-1}\big[\frac{1}{\Phi}+\nu'_{g'}\big].$$

We can summarize the result in terms of a proposition:

Proposition 4. *("Complete markets without commitment") The excess burden of taxation follows ,*

$$\frac{1}{\Phi_{t+1}} = [1+\frac{u_{cc,t+1}-u_{cl,t+1}}{u_{c,t+1}}\mathcal{C}_b b_{t+1}]^{-1}\big[\frac{1}{\Phi_t}+\nu_{t+1}\big] \tag{39}$$

with $\nu_{t+1} = -[A(V_{t+1})u_{c,t+1}b_{t+1} - A(\mu_t) \cdot E_t m_{t+1} u_{c,t+1} b_{t+1}]$.

Note that the marginal utility channel always commands to put more distortions for the future, as long there is issuance of debt, $b'_{g'} > 0$. On the other hand, the continuation value channel commands more distortions when $\nu'_{g'} < 0$ (high debt positions) and less distortions when $\nu'_{g'} > 0$ (low debt positions). So if the planner insures against the bad times wiht low debt positions, then we have two opposing incentives in bad times: tax less to increase prices through the value channel because you have a negative relative position, but tax more in order to increase prices, because your gross position is positive (coming from the Markov-perfect assumption). In good times, the two incentives align with each other. So it seems that there will be more taxation in good times (relative to MPE and time-additive utility) and less taxation (relative to MPE with time-additive utility) in bad times.

5.3 Recursive utility and incomplete markets

Consider now the case of incomplete markets with recursive utility and lack of commitment. To see the analysis in the time-additive case consult Karantounias (2017). In contrast to the case of incomplete markets and commitment, we do not need to formulate the problem *before* the realization of uncertainty at time t (which was necessary in order to capture the promises *across states* of the planner).

Let (b_-, g) denote the state variable, where b_- corresponds to the state non-contingent debt at the beginning of the period. Let $\mathcal{C}$ and $\mathcal{H}$ denote the consumption and labor policy function of the future policy maker. Let $V(b_-, g)$ denote the value function at (b_-, g). The Bellman equation takes the form

$$V(b_-, g) = \max_{c \geq 0, h \in [0,1], b \in \mathcal{B}} u(c, 1-h) + \beta H^{-1}\Big(\sum_{g'} \pi(g'|g) H(V(b, g'))\Big)$$

subject to

$$u_c(c, 1-h)b_- = u_c(c, 1-h)c - u_l(c, 1-h)h + \beta\Big(\sum_{g'} \pi(g'|g) m'_{g'} u_c(\mathcal{C}(b, g'), 1 - \mathcal{H}(b, g'))\Big)b$$

$$c + g = h$$

where $m'_{g'} \equiv \frac{H'\big(V(b,g')\big)}{H'\Big(H^{-1}\big(\sum_{g'} \pi(g'|g) H(V(b,g'))\big)\Big)}$. The optimal policy functions will be functions of (b_-, g), so

$$
\begin{aligned}
c &= c(b_-, g) \\
h &= h(b_-, g) \\
b &= b(b_-, g)
\end{aligned}
$$

The Markov-perfect time-consistency requirement is that $c(b_-, g) = \mathcal{C}(b_-, g), h(b_-, g) = \mathcal{H}(b_-, g)$ $\forall(b_-, g)$ in the state space. Assign multipliers Φ and λ on the implementability and resource constraint respectively and define the *revenue* (in marginal utility units) as

$$
R(b, g) \equiv (\sum_{g'} \pi(g'|g) m'_{g'} u'_c) \cdot b = \frac{\sum_{g'} \pi(g'|g) H'(V(b, g')) u_c(\mathcal{C}(b, g'), 1 - \mathcal{H}(b, g'))}{H'(H^{-1}(\sum_{g'} \pi(g'|g) H(V(b, g'))))} \cdot b \tag{40}
$$

The first-order necessary conditions take the form

$$
\begin{aligned}
c: &\quad u_c + \Phi[\Omega_c - u_{cc} b_-] = \lambda \\
h: &\quad u_l - \Phi[\Omega_h + u_{cl} b_-] = \lambda \\
b: &\quad \sum_{g'} \pi(g'|g) m'_{g'} V_b(b, g') + \Phi \frac{\partial R}{\partial b} = 0
\end{aligned}
$$

As usual, note that the optimality condition with respect to b takes the typical MC and MR formulation of (1). Note also that the envelope condition is

$$
V_b(b_-, g) = -\Phi u_c.
$$

The marginal revenue formula will depict the marginal utility and the continuation utility channel. In contrast to the analysis with complete markets, debt here is non-contingent, so the *average* effect across states is the one that matters.

Proposition 5. *("Incomplete markets without commitment") The excess burden of taxation without commitment and incomplete market follows the law of motion (in sequential notation)*

$$
E_t n_{t+1} \Phi_{t+1} (1 - \xi_{t+1} b_t \Phi_t) = \Phi_t \big[1 + b_t \cdot E_t n_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \mathcal{C}_{b,t+1}\big] \tag{41}
$$

where $\xi_{t+1} \equiv A(V_{t+1}) u_{c,t+1} - A(\mu_t) E_t m_{t+1} u_{c,t+1}$, the relative marginal utility position.

Proof. See the Appendix. □

Note that we cannot divide any more as in the complete markets case so we cannot express anymore the law of motion in terms of the *inverse* excess burden of taxation. Furthermore, note that if $b_t > 0$ and that if we have $E_t^n \Phi_{t+1}\xi_{t+1} > 0$ (which may be happening if we tax more the bad times, and these times have also high marginal utility), then recursive utility will *amplify* again the efforts of the Markov-perfect policy guy.

Proposition 6. *("Tax rate without commitment")*

- *Assume that markets are* <u>*complete*</u>*. The optimal tax rate without commitment for* $t \geq 0$ *is*

$$\tau_t = \frac{\Phi_t\big[(\epsilon_{cc,t} + \epsilon_{hc,t})(1 - \frac{b_t}{c_t}) + \epsilon_{ch,t} + \epsilon_{hh,t}\big]}{1 + \Phi_t\big(1 + \epsilon_{hh,t} + \epsilon_{hc,t}(1 - \frac{b_t}{c_t})\big)}, \tag{42}$$

where the excess burden of taxation follows the law of motion (39).

- *Assume that markets are* <u>*incomplete*</u>*. The respective tax rate for* $t \geq 0$ *is*

$$\tau_t = \frac{\Phi_t\big[(\epsilon_{cc,t} + \epsilon_{hc,t})(1 - \frac{b_{t-1}}{c_t}) + \epsilon_{ch,t} + \epsilon_{hh,t}\big]}{1 + \Phi_t\big(1 + \epsilon_{hh,t} + \epsilon_{hc,t}(1 - \frac{b_{t-1}}{c_t})\big)}, \tag{43}$$

where the excess burden of taxation satisfies the law of motion (41).

Proof. See the Appendix. The proof uses the first-order conditions with respect to (c, h). □

6 Concluding remarks

In this paper I extend the literature on dynamic fiscal policy with a representative agent using a common guiding principle about the relative costs and benefits of distortionary taxes across states and dates. The government manipulates the returns of the government portfolio in order to minimize the welfare costs of taxes. Government debt return manipulation depends on pricing kernels, market structure and commitment protocol, generating therefore differing policy prescriptions, which are manifestations of the *same* principle though: levy more taxes on a state (date) if it is cheaper to issue debt against this state (date). This simple economic principle goes a long way in interpreting and extending the theory of optimal taxation in setups with recursive preferences that match better asset pricing facts.

A Optimal tax rate in proposition 1

The derivation of the tax rate in (25) uses the first-order conditions with respect to (c, h) and is provided here for completeness. The derivation in the incomplete markets case of proposition 3 follows similar steps, so the proof for the complete markets case serves also as a roadmap. Let

$$\Omega(c, h) \equiv u_c(c, 1-h)c - u_l(c, 1-h)h. \tag{A.1}$$

Ω stands for the government surplus in marginal utility units as a function of the allocation (c, h). Let $\Omega_i, i = c, h$ stand for the respective partial derivative and let λ denote the multiplier on the resource constraint (20). The first-order conditions with respect to consumption and labor are

$$c: \quad u_c + \Phi\Omega_c = \lambda \tag{A.2}$$
$$h: \quad u_l - \Phi\Omega_h = \lambda \tag{A.3}$$

Combine (A.2) and (A.3) in order to eliminate λ to get $\frac{u_l}{u_c} \cdot \frac{1-\Phi\Omega_h/u_l}{1-\Phi\Omega_c/u_c} = 1$. Note that

$$\frac{\Omega_c}{u_c} = 1 - \epsilon_{cc} - \epsilon_{ch} \tag{A.4}$$
$$\frac{\Omega_h}{u_l} = -1 - \epsilon_{hh} - \epsilon_{hc}, \tag{A.5}$$

where the elasticities defined in proposition 1. Use the fact that $u_l/u_c = 1 - \tau$ and rewrite the combined condition in terms of the tax rate in order to get (25).

B Initial period problems

Here I state the problem at $t = 0$ for the commitment case when we have complete or incomplete markets. These problems determine the initial allocation of consumption and labor and the *optimal* initial value of the pseudo-state variables z and B respectively.

B.1 Complete markets

The planner at $t = 0$ chooses $\{c_0, h_0\}$ and $z_{1,g_1} \forall g_1$ to maximize

$$u(c_0, 1-h_0) + \beta H^{-1}\Big(\sum_{g_1} \pi(g_1|g_0) H\big(V(z_{1,g_1}, g_1)\big)\Big)$$

subject to

$$
\begin{aligned}
&u_c(c_0, 1-h_0)b_0 = u_c(c_0, 1-h_0)c_0 - u_l(c_0, 1-h_0)h_0 + \beta \sum_{g_1} \pi(g_1|g_0) m_{1,g_1} z_{1,g_1} \\
&c_0 + g_0 = h_0 \\
&c_0 \geq 0, h_0 \in [0,1], z_{1,g_1} \in Z(g_1),
\end{aligned}
$$

where (b_0, g_0) are given. As usual, m_{1,g_1} is shorthand for the continuation value adjustment in the stochastic discount factor, $m_{1,g_1} = H'(V(z_{1,g_1}, g_1))/H'(\mu_0)$, where $\mu_0 = H^{-1}(\sum_{g_1} \pi(g_1|g_0) H\big(V(z_{1,g_1}, g_1)\big))$. The notation z_{1,g_1} captures that z_1 is contingent on the realization of the shock at $t = 1$.

B.2 Incomplete markets

The planner chooses $c_0 \geq 0$, $h_0 \in [0,1]$ and $B_0 \in [\underline{B_0}, \bar{B}_0]$ to maximize

$$
u(c_0, 1-h_0) + \beta W(B_0, g_0)
$$

subject to

$$
\begin{aligned}
&u_c(c_0, 1-h_0)b_{-1} = u_c(c_0, 1-h_0)c_0 - u_l(c_0, 1-h_0)h_0 + \beta B_0 \\
&c_0 + g_0 = h_0,
\end{aligned}
$$

where b_{-1} and the initial shock g_0 are given.

C Incomplete markets under commitment

It is convenient to keep the random variable m_g and treat (32) as an additional constraint. Assign multipliers $\pi(g|g_-)\tilde{\Phi}_g$, $\pi(g|g_-)\tilde{\lambda}_g$ and $\pi(g|g_-)\zeta_g$ on constraints (29), (30) and (32) respectively and recall the definition of Ω in (A.1). The Lagrangian takes the form

$$
\begin{aligned}
L = & H^{-1}\Big(\sum_g \pi(g|g_-)H\big(u(c_g,1-h_g)+\beta W(B_g,g)\big)\Big) \\
& -\sum_g \pi(g|g_-)\tilde{\Phi}_g\Big[\frac{u_c(c_g,1-h_g)}{\sum_g \pi(g|g_-)m_g u_c(c_g,1-h_g)}B_- - \Omega(c_g,h_g) - \beta B_g\Big] \\
& -\sum_g \pi(g|g_-)\tilde{\lambda}_g[c_g+g-h_g] \\
& -\sum_g \pi(g|g_-)\zeta_g\Big[m_g - \frac{H'(u(c_g,1-h_g)+\beta W(B_g,g))}{H'\Big(H^{-1}\big(\sum_g \pi(g|g_-)H(u(c_g,1-h_g)+\beta W(B_g,g))\big)\Big)}\Big].
\end{aligned}
$$

Define

$$
K \equiv \frac{\sum_g \pi(g|g_-)u_c(c_g,1-h_g)\tilde{\Phi}_g}{\sum_g m_g u_c(c_g,1-h_g)} \quad \text{and} \quad I \equiv \frac{\sum_g \pi(g|g_-)H'\big(u(c_g,1-h_g)+\beta W(B_g,g)\big)\zeta_g}{H'\Big(H^{-1}\big(\sum_g \pi(g|g_-)H(u(c_g,1-h_g)+\beta W(B_g,g))\big)\Big)}.
$$

C.1 First-order conditions

The first-order necessary conditions for an interior solution take the form:

$$
\begin{aligned}
c_g: &\quad \pi(g|g_-)m_g u_c(c_g,1-h_g)+\pi(g|g_-)\tilde{\Phi}_g\Omega_c(c_g,h_g)+\frac{\partial I}{\partial c_g}-\frac{\partial K}{\partial c_g}B_- = \pi(g|g_-)\tilde{\lambda}_g && \text{(C.1)} \\
h_g: &\quad -\pi(g|g_-)m_g u_l(c_g,1-h_g)+\pi(g|g_-)\tilde{\Phi}_g\Omega_h(c_g,h_g)+\frac{\partial I}{\partial h_g}-\frac{\partial K}{\partial h_g}B_- = -\pi(g|g_-)\tilde{\lambda}_g && \text{(C.2)} \\
m_g: &\quad \pi(g|g_-)\zeta_g = -\frac{\partial K}{\partial m_g}B_- && \text{(C.3)} \\
B_g: &\quad -\beta\pi(g|g_-)m_g W_B(B_g,g) = \beta\pi(g|g_-)\tilde{\Phi}_g + \frac{\partial I}{\partial B_g} && \text{(C.4)}
\end{aligned}
$$

The derivatives of the expression I denote the continuation value effects on the pricing of debt where the derivatives of K the effect of the marginal utility of consumption, leisure and the term m on the pricing of legacy debt B_-. We have

$$
\begin{aligned}
\frac{\partial K}{\partial m_g} &= -\pi(g|g_-)x_g\sum_g \pi(g|g_-)x_g\tilde{\Phi}_g && \text{(C.5)} \\
\frac{\partial K}{\partial c_g} &= \pi(g|g_-)\frac{u_{cc}(c_g,1-h_g)}{\sum_g \pi(g|g_-)m_g u_c(c_g,1-h_g)}\big[\tilde{\Phi}_g - m_g\sum_g \pi(g|g_-)x_g\tilde{\Phi}_g\big] && \text{(C.6)} \\
\frac{\partial K}{\partial h_g} &= -\pi(g|g_-)\frac{u_{cl}(c_g,1-h_g)}{\sum_g \pi(g|g_-)m_g u_c(c_g,1-h_g)}\big[\tilde{\Phi}_g - m_g\sum_g \pi(g|g_-)x_g\tilde{\Phi}_g\big] && \text{(C.7)}
\end{aligned}
$$

and

$$\frac{\partial I}{\partial c_g} = -\pi(g|g_-)m_g u_c(c_g, 1-h_g)\nu_g \tag{C.8}$$

$$\frac{\partial I}{\partial h_g} = \pi(g|g_-)m_g u_l(c_g, 1-h_g)\nu_g \tag{C.9}$$

$$\frac{\partial I}{\partial B_g} = -\beta\pi(g|g_-)m_g W_B(B_g, g)\nu_g \tag{C.10}$$

where

$$\nu_g \equiv A(V_g)\zeta_g - A(\mu_-)\sum_g \pi(g|g_-)m_g\zeta_g, \tag{C.11}$$

the "innovation" (adjusted properly by absolute risk aversion) in the multiplier ζ_g, which captures the shadow value of increasing m_g. V_g is shorthand for value at g, $V_g = u(c_g, 1-h_g) + \beta W(B_g, g)$ and μ_- denotes the certainty equivalent at $t-1$, so at the optimum it is equal to $W(B_-, g_-)$.

Use now (C.5) in (C.3) to get

$$\zeta_g = x_g\big(\sum_g \pi(g|g_-)x_g\tilde{\Phi}_g\big)B_- = x_g\big(\sum_g \pi(g|g_-)n_g\Phi_g\big)B_-, \tag{C.12}$$

where we used the *normalized* multiplier $\Phi_g \equiv \tilde{\Phi}_g/m_g$ and the definition of the change of measure n_g from the text, $n_g \equiv m_g x_g$, with $\sum_g \pi(g|g_-)n_g = 1$. Note that we have

$$\sum_g \pi(g|g_-)m_g\zeta_g = \big(\sum_g \pi(g|g_-)n_g\Phi_g\big)B_-. \tag{C.13}$$

Thus, the innovation ν_g in (C.11) becomes

$$\nu_g = \big[A(V_g)x_g - A(\mu_-)\big]\big(\sum_g \pi(g|g_-)n_g\Phi_g\big)B_-g = \xi_g \cdot \big(\sum_g \pi(g|g_-)n_g\Phi_g\big)b_- \tag{C.14}$$

where ξ_g defined in (34) in the text, and b_- stands for debt in consumption units, $b_- \equiv \frac{B_-}{\sum_g \pi(g|g_-)m_g u_c(c_g, 1-h_g)}$.

C.2 Excess burden of taxation

Use now (C.10) in (C.4), simplify, and use (C.14) to get (33) in the text. Moreover, we can collect terms to get

$$-W_B(B_g, g)(1-\nu_g) = \Phi_g \Rightarrow \frac{1}{-W_B(B_g, g)} = \frac{1}{\Phi_g} - \frac{\nu_g}{\Phi_g} \tag{C.15}$$

Use now (C.14) to finally get

$$\frac{1}{-W_B(B_g, g)} = \frac{1}{\Phi_g} - \frac{\sum_g \pi(g|g_-) n_g \Phi_g}{\Phi_g} \xi_g b_-. \tag{C.16}$$

The envelope condition is

$$W_B(B_-, g_-) = -\sum_g \pi(g|g_-) x_g \tilde{\Phi}_g = -\sum_g \pi(g|g_-) n_g \Phi_g. \tag{C.17}$$

Update one period, turn into sequence notation and replace $W_B(B_g, g)$ in (C.16) to get the law of motion (35) in the text.

Initial period. Note that (C.15) holds from period one onward, i.e. $-W_B(B_t, g_t)(1-\nu_t) = \Phi_t$ for $t \geq 1$. Assign multiplier Φ_0 on the implementability constraint at the initial period problem in section B.2 to get the first-order condition $-W(B_0, g_0) = \Phi_0$. From the envelope condition of the problem from period one onward in (C.17) we have $W_B(B_0, g_0) = -E_0 n_1 \Phi_1$. Combining these two conditions we get $E_0 n_1 \Phi_1 = \Phi_0$. In the law of motion (35), this is achieved by setting $\xi_0 \equiv 0$.

C.3 Proof of proposition 3

Proof. Use (C.6) and (C.8) in (C.1), define the normalized multiplier $\lambda_g \equiv \tilde{\lambda}_g / m_g$, simplify and collect terms to get the final version of the optimality condition with respect to c:

$$u_c(c_g, 1-h_g)(1-\nu_g) + \Phi_g \Omega_c(c_g, h_g) - u_{cc}(c_g, 1-h_g)\big[\Phi_g - \sum_g \pi(g|g_-) n_g \Phi_g\big] b_- = \lambda_g \tag{C.18}$$

Similarly, use (C.7) and (C.9) in (C.2) to get

$$u_l(c_g, 1-h_g)(1-\nu_g) - \Phi_g \Omega_h(c_g, h_g) - u_{cl}(c_g, 1-h_g)\big[\Phi_g - \sum_g \pi(g|g_-) n_g \Phi_g\big] b_- = \lambda_g. \tag{C.19}$$

Combine now (C.18) and (C.19) and eliminate λ_g to get

$$\frac{u_l}{u_c}\cdot\frac{1-\nu_g-\Phi_g\frac{\Omega_h}{u_l}-\frac{u_{cl}\cdot c}{u_l}(\Phi_g-\sum_g \pi(g|g_-)n_g\Phi_g)\frac{b_-}{c}}{1-\nu_g+\Phi_g\frac{\Omega_c}{u_c}-\frac{u_{cc}\cdot c}{u_c}(\Phi_g-\sum_g \pi(g|g_-)n_g\Phi_g)\frac{b_-}{c}}=1, \tag{C.20}$$

where I suppress the arguments of the various functions in order to ease notation. Recall that $u_l/u_c = 1-\tau$. Use the elasticity expressions in (A.4) and (A.5), rewrite (C.20) in terms of τ and turn into sequence notation to get expression (36) in the text.

□

D No commitment

D.1 Proof of proposition 6

The marginal revenue takes the form

$$\frac{\partial R(b,g)}{\partial b}=\sum_{g'}\pi(g'|g)m'_{g'}u'_c+b\cdot\{\sum_{g'}\pi(g'|g)m'_{g'}(u'_{cc}-u'_{cl})\mathcal{C}_b-\sum_{g'}\pi(g'|g)m'_{g'}V_b(b,g')\xi'_{g'}\}$$

where $\xi'_{g'}\equiv A(V(b,g'))u'_c-A(\mu)\sum_{g'}\pi(g'|g)m'_{g'}u'_c$. Use now the envelope condition and the expression for the marginal revenue and rewrite the optimality condition with respect to debt as

$$\sum_{g'}\pi(g'|g)m'_{g'}u'_c\Phi'_{g'}(1-\xi'_{g'}b\Phi)=\Phi\big[\sum_{g'}\pi(g'|g)m_{g'}u'_c+b\cdot\sum_{g'}\pi(g'|g)m'_{g'}(u'_{cc}-u'_{cl})\mathcal{C}_b\big]$$

Divide over $\sum_{g'}\pi(g'|g)m_{g'}u'_c$ and express in terms of the n measure as

$$\sum_{g'}\pi(g'|g)n'_{g'}\Phi'_{g'}(1-\xi'_{g'}b\Phi)=\Phi\big[1+b\cdot\sum_{g'}\pi(g'|g)n'_{g'}(\frac{u'_{cc}-u'_{cl}}{u'_c})\mathcal{C}_b\big]$$

The result follows.

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