A Fiscal Theory of Trend Inflation

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- We build and estimate a TANK model with partially **unfunded** government debt:
  1. Business cycle and monetary policy shocks propagate as usual
  2. **Unfunded** fiscal shocks generate movements in trend inflation that the central bank accommodates $\Rightarrow$ **a fiscal theory of trend inflation**

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- **Fiscal theory trend inflation** accounts for the bulk of the inflation dynamics:
  1. A persistent and **partially unfunded** rise in transfers in the mid-1960s (*Great Society*) accounts for the increase in trend inflation during the Great Inflation
  2. Partially unfunded debt has offset the deflationary bias from the 1990s and on
  3. Recent sizable fiscal stimuli require policy coordination to avoid rise in trend inflation

What Does This Theory Predict for the Post-pandemic?

- **Fiscal vulnerability and monetary policy**: When spending is large, **beliefs** about what share of debt is unfunded may lead to large swings in trend inflation
- **Historically**, this share has been moving **sluggishly** in the US, but **the future can be different from the past**
- **Monetary and fiscal policy coordination**:
  1. The fiscal authority needs a credible plan to stabilize the fraction of fiscally funded debt compatible with the central bank's target
  2. The central bank credibly committed to limit inflation deviations from this target
  3. Heightened geopolitical risk may reduce the deflationary bias, requiring a reduction in the share of unfunded debt

A TANK Model with Partially Unfunded Debt

The Model

State-of-the-art TANK model

- Distortionary taxation on labor and capital income
- Hand-to-mouth households
- Long-term government bonds
- **Typical set of business cycle shocks** plus **fiscal shocks** and a **shifter of the Phillips curve** capturing market and non policy forces such as globalization and demographic changes

Underfunded Debt and Monetary and Fiscal Coordination

- Two types of transfers:

  1. **Funded** transfers: Transfers backed by future fiscal adjustments
     $\Rightarrow$ **Monetary-led** policy mix

  2. **Unfunded** transfers: Transfers **not** backed by future fiscal adjustments
     $\Rightarrow$ **Fiscally-led** policy mix

- The monetary authority **tolerates** the increase in inflation needed to stabilize the resulting amount of **unfunded** debt

Fiscal and Monetary Rules

Fiscal Rules

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1-\rho_G)\, \gamma_G \tilde{b}^M_{t-1} + \zeta_{g,t}$$

$$\hat{z}_t = \phi_{zy}\hat{y}_t + \rho_Z \hat{z}_{t-1} - (1-\rho_Z)\, \gamma_Z \tilde{b}^M_{t-1} + \zeta^M_{z,t} + \zeta^F_{z,t}$$

$$\hat{\tau}^L_t = \rho_L \hat{\tau}^L_{t-1} + (1-\rho_L)\, \gamma_L \tilde{b}^M_{t-1} + \zeta_{\tau_L,t}$$

$$\hat{\tau}^K_t = \rho_K \hat{\tau}^K_{t-1} + (1-\rho_K)\, \gamma_K \tilde{b}^M_{t-1} + \zeta_{\tau_K,t}$$

Monetary Rule

$$\hat{R}_t = \max\left(-\ln R_*, \rho_r \hat{R}_{t-1} + (1-\rho_r)\left[\phi_\pi\left(\hat{\pi}_t - \hat{\pi}^F_t\right) + \phi_y \hat{y}_t\right]\right) + \epsilon_{R,t}$$

Definition of Funded Debt and the Inflation Target

- The funded share of debt $\tilde{b}_t^M$ is stabilized by fiscal instruments
  1. The parameters γ_G, γ_Z, γ_L, and γ_K are sufficiently large to back the funded debt $\tilde{b}_t^M$
  2. Changes in transfers $\zeta_{z,t}^F$ determine the share of unfunded debt

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- The inflation target, $\hat{\pi}_t^F$, is the increase in inflation needed to stabilize the unfunded share of the debt $\left(\tilde{b}_t - \tilde{b}_t^M\right)$
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- The funded debt and the inflation target are defined using a shadow economy

Constructing the Shadow Economy

Monetary-led policy mix in the shadow economy

→ Shocks to unfunded transfers $\zeta^F_{z,t}$ are shut down and the whole public debt $\tilde{b}^M_t$ in the shadow economy is funded

→ Taylor principle is satisfied: Response to $\hat{\pi}^M_t$ more than one-to-one

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Fiscally-led policy mix in response to the unfunded debt

- → Debt in the actual economy is $\tilde{b}_t > \tilde{b}^M_t$
- → The inflation target in the actual economy is

$$\hat{\pi}^F_t \equiv \hat{\pi}_t - \hat{\pi}^M_t$$

which is the change in inflation needed to stabilize the amount of unfunded debt

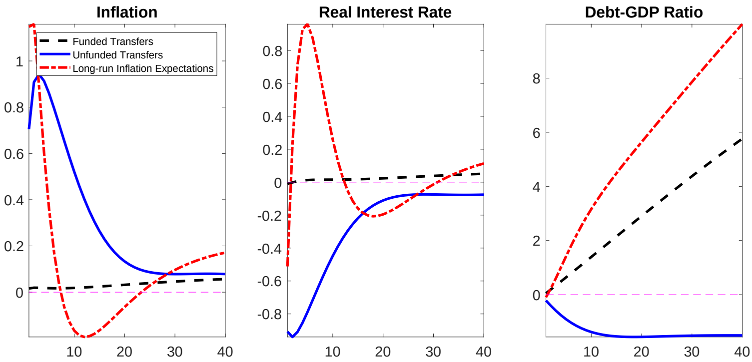
Empirical Analysis

Estimation

- The model is estimated using a data set of 20 macro and fiscal variables
  1. Real GDP growth
  2. Real consumption growth
  3. Real investment growth
  4. Hours worked
  5. Inflation (GDP deflator)
  6. Growth rate of real average weekly earnings
  7. Real transfers payments growth rate
  8. Real government consumption and investment growth rate
  9. Debt to GDP ratio
  10. Federal funds rate (FFR)

  11-20. 1Q-10Q ahead expected market path of the FFR (OIS data)
- Sample periods: 1960q1-2007q4 and 2008q1-2020q4
- Second sample includes all the 20 observables; re-estimation of standard deviations and the factor model governing the forward guidance shocks (Campbell et al. 2012)

Identification of Unfunded Transfers Shocks

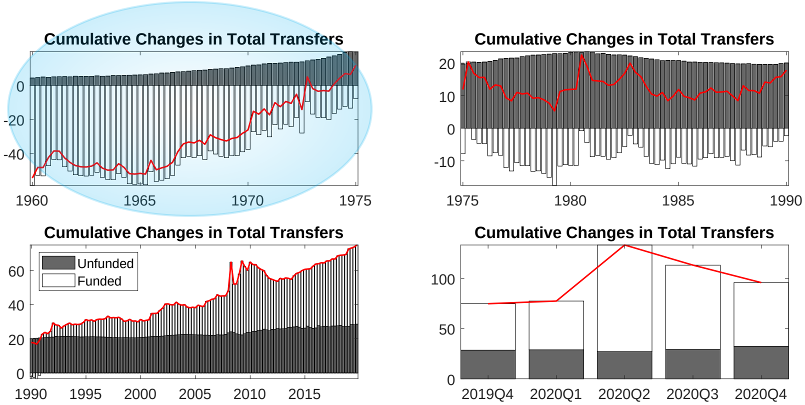


- **Funded transfers**: **Modest** impact on the macroeconomy, debt **increase**
- **Unfunded transfers**: **Persistent** inflation increase, real rate decline, debt **decline**
- **Phillips curve shifter**: **Temporary** inflation spike, real rate increase, debt **increase**

A Fiscal Theory of Trend Inflation

- Shocks to the **unfunded portion of government debt** are accommodated by the central bank
- These shocks lead to a **persistent** increase in inflation and inflation expectations
- **Identification** of these shocks rests on the joint dynamics of inflation, inflation expectations, real interest rates, and the debt-to-GDP ratio

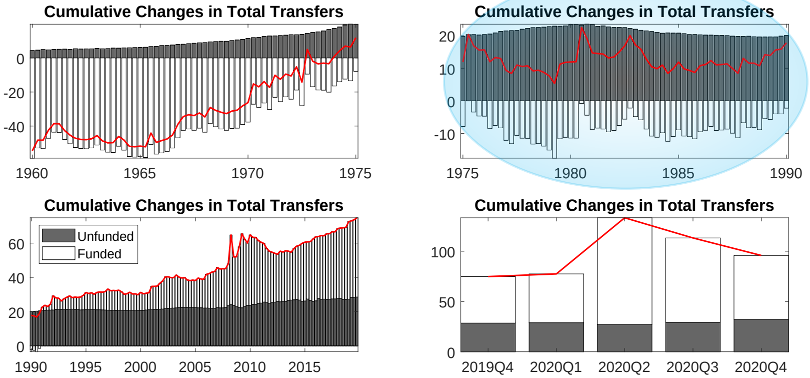
Funded and Unfunded Transfers



Four phases:

1. From the 1960s to the mid-1970s: Large rise of unfunded transfers

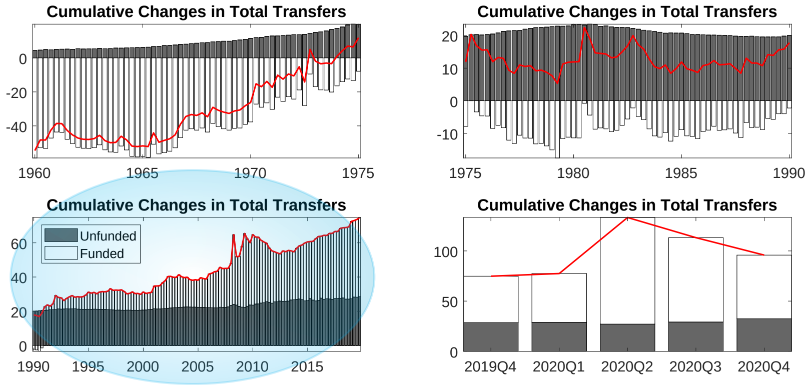
Funded and Unfunded Transfers



Four phases:

2. From the mid-1970s to the 1990s: Stability, with hump shape in unfunded transfers

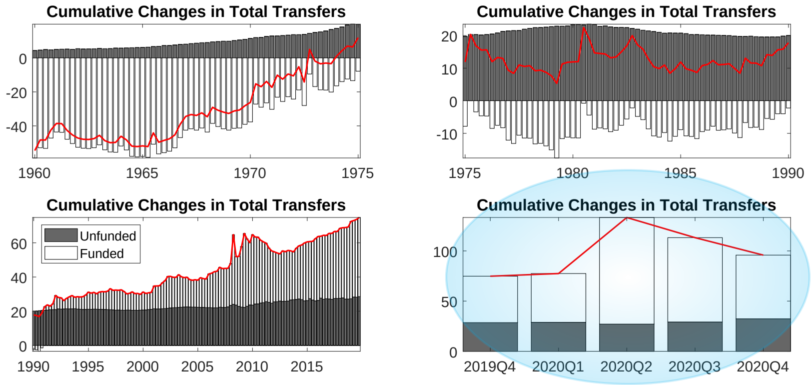
Funded and Unfunded Transfers



Four phases:

3. From the 1990s to the Pandemic: Further rise, predominantly funded

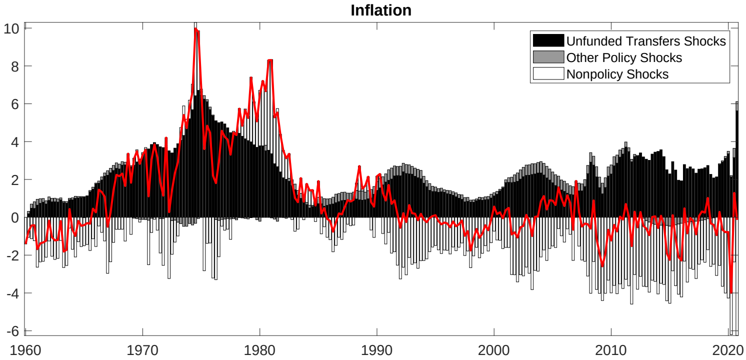
Funded and Unfunded Transfers



Four phases:

4. The COVID stimulus package

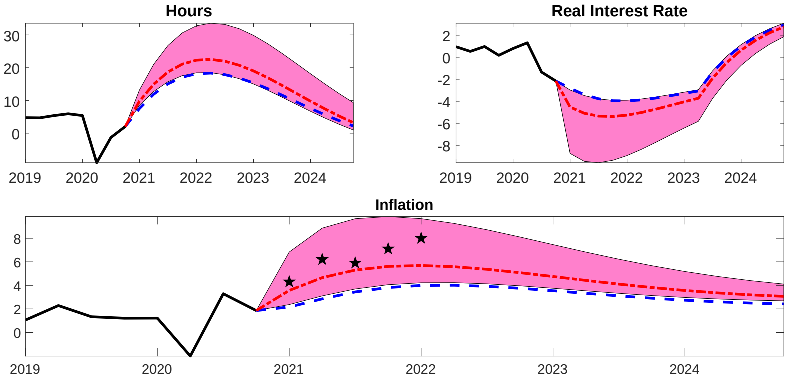
Drivers of Inflation



Unfunded spending:

1. Accounts for rise of trend inflation in the 1960s-1970s and decline in the 1980s
2. Offsets the deflationary bias that non-policy shocks have set off since early 1990s

ARPA Fiscal Stimulus and Inflation



Baseline: Forecast based on filtered data up to 2020Q4

Counterfactual: Forecast including ARPA shock based on transfer payments in 2021Q1 attributed to funded and unfunded transfers according to historical pattern ▸ Scenarios

Concluding Remarks

- **Fiscal vulnerability and monetary policy**: When spending is large, **beliefs** about what share of spending is unfunded may lead to large swings in trend inflation
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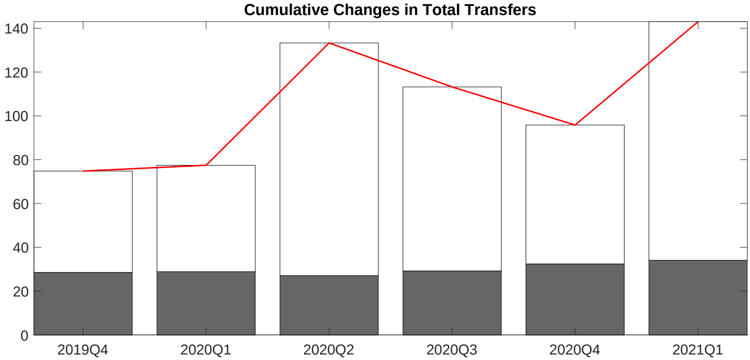
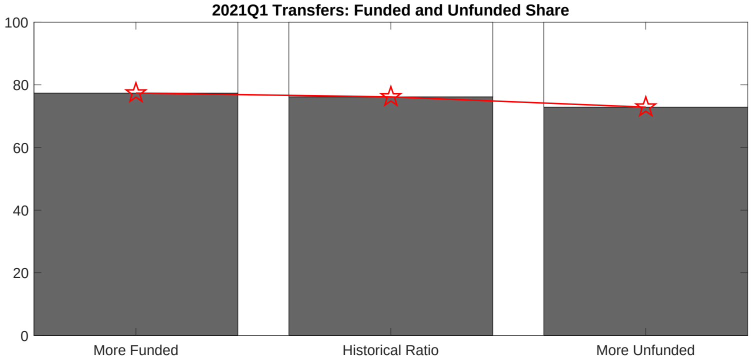
Appendix

Funded and Unfunded Transfers (2020q1-2021q1)



Three Scenarios for the ARPA Transfers



Back

Calibrated Parameters

Parameters Fixed in Estimation

	Parameters	Values
Discount factor	β	0.9900
Debt maturity decay rate	ρ	0.9680
Capital depreciation rate	δ	0.0250
Elasticity of output to capital	α	0.3300
Wage markup	η_w	0.1400
Price markup	η_p	0.1400
Government expenditures to GDP ratio	s_{gc}	0.1100
Steady state tax rate on labor income	τ_L	0.1860
Steady state tax rate on capital income	τ_K	0.2180
Steady state tax rate on consumption	τ_C	0.0230

First Sample Estimates

Prior and Posterior Distribution for Structural Parameters

	Posterior Distribution				Prior Distribution		
Param	Mode	Median	5%	95%	Type	Mean	Std
s_b	2.1703	2.1834	2.0147	2.3497	N	1.8200	0.1000
$100\ln\mu$	0.4000	0.4001	0.3255	0.4925	N	0.5000	0.0500
$100\ln\Pi$	0.5402	0.5195	0.4267	0.6104	N	0.5000	0.0500
ξ	1.9704	1.9167	1.7493	2.1217	N	2.0000	0.2500
μ	0.0771	0.0778	0.0652	0.0925	N	0.1100	0.0100
ω_w	0.8041	0.8063	0.7861	0.8243	B	0.5000	0.1000
ω_p	0.8663	0.8666	0.8375	0.8897	B	0.5000	0.1000
ψ	0.6596	0.6572	0.5755	0.7502	B	0.5000	0.1000
s	5.7144	5.5214	5.0185	5.9213	N	6.0000	0.5000
χ_w	0.0372	0.0437	0.0164	0.0923	B	0.5000	0.2000
χ_p	0.3117	0.2782	0.1279	0.4101	B	0.5000	0.2000
θ	0.9106	0.9091	0.8985	0.9187	B	0.5000	0.2000
α^G	-0.0455	-0.0396	-0.1832	0.0838	N	0.0000	0.1000

First Sample Estimates

Prior and Posterior Distribution

	Posterior Distribution				Prior Distribution		
Param	Mode	Median	5%	95%	Type	Mean	Std
ϕ_y	0.0012	0.0019	0.0001	0.0074	N	0.2500	0.1000
ϕ_π	2.0577	2.0963	1.9462	2.2525	N	2.0000	0.1000
ϕ_{zy}	0.0715	0.0439	0.0198	0.0719	G	0.1000	0.0500
γ_G	0.3800	0.3463	0.2218	0.4279	N	0.2500	0.1000
γ_K	0.0043	0.0064	0.0003	0.0335	N	0.2500	0.1000
γ_L	0.0163	0.0133	0.0009	0.0461	N	0.2500	0.1000
γ_Z	0.0017	0.0063	0.0003	0.0249	N	0.2500	0.1000
ρ_r	0.7250	0.7223	0.6650	0.7746	B	0.5000	0.1000
ρ_G	0.9637	0.9627	0.9340	0.9803	B	0.5000	0.1000
ρ_Z	0.5007	0.4313	0.3430	0.5448	B	0.5000	0.1000
ρ_K	0.5000	0.4690	0.3798	0.5586	B	0.5000	0.1000
ρ_L	0.4977	0.5015	0.3873	0.6409	B	0.5000	0.1000
ρ_C	0.4996	0.4280	0.3698	0.4818	B	0.5000	0.1000

First Sample Estimates

Prior and Posterior Distribution

	Posterior Distribution				Prior Distribution		
Param	Mode	Median	5%	95%	Type	Mean	Std
ρ_{eG}	0.2868	0.3045	0.1506	0.3782	B	0.5000	0.1000
ρ_{eZ}^{M}	0.9954	0.9953	0.9933	0.9968	B	0.9950	0.0010
ρ_{eZ}^{F}	0.9958	0.9956	0.9937	0.9971	B	0.9950	0.0010
ρ_{a}	0.2987	0.2803	0.1711	0.3610	B	0.5000	0.1000
ρ_{b}	0.8237	0.8237	0.7774	0.8609	B	0.5000	0.1000
ρ_{em}	0.2407	0.2573	0.1692	0.3105	B	0.5000	0.1000
ρ_{i}	0.9205	0.9206	0.8990	0.9395	B	0.5000	0.1000
ρ_{rp}	0.9085	0.9062	0.8880	0.9220	B	0.5000	0.1000
$\rho_{\pi NKPC}$	0.9965	0.9966	0.9951	0.9977	B	0.9950	0.0010

First Sample Estimates

Prior and Posterior Distribution							
	Posterior Distribution				Prior Distribution		
Param	Mode	Median	5%	95%	Type	Mean	Std
σ_G	1.9046	1.9306	1.7416	2.1419	IG	0.5000	0.2000
σ_Z^M	2.9635	2.8922	2.6631	3.0924	IG	0.5000	0.2000
σ_Z^F	0.5166	0.5500	0.4194	0.7319	IG	0.1000	0.0500
σ_a	1.2113	1.1989	1.0895	1.3349	IG	0.5000	0.2000
σ_b	4.9850	4.9782	4.9214	4.9986	IG	0.2500	0.2000
σ_m	0.2375	0.2406	0.2154	0.2691	IG	0.5000	0.2000
σ_i	0.5192	0.5318	0.4734	0.5955	IG	0.5000	0.2000
σ_w	0.3487	0.3512	0.3156	0.3912	IG	0.5000	0.2000
σ_p	0.1625	0.1640	0.1427	0.1877	IG	0.5000	0.2000
σ_{rp}	0.3914	0.3990	0.3441	0.4586	IG	0.5000	0.2000
$\sigma_{\pi NKPC}$	1.3255	1.3763	1.2106	1.6382	IG	0.1000	0.0500
σ_{GDP}^m	0.4330	0.4352	0.3947	0.4831	IG	0.5000	0.2000
σ_{by}^m	0.3160	0.3032	0.2221	0.4217	IG	0.5000	0.2000

Second Sample Estimates

Prior and Posterior Distribution: Second sample							
	Posterior Distribution				Prior Distribution		
Param	Mode	Median	5%	95%	Type	Mean	Std
σ_G	3.2021				IG	0.5000	0.2000
σ_Z^M	4.9982				IG	0.5000	0.2000
σ_Z^F	1.0214				IG	0.1000	0.0500
σ_a	3.7944				IG	0.5000	0.2000
σ_b	4.9975				IG	0.2500	0.2000
σ_m	0.1242				IG	0.5000	0.2000
σ_i	2.5281				IG	0.5000	0.2000
σ_w	0.6567				IG	0.5000	0.2000
σ_p	0.1630				IG	0.5000	0.2000
σ_{rp}	2.8727				IG	0.5000	0.2000
$\sigma_{\pi NKPC}$	4.9939				IG	0.1000	0.0500
σ_{GDP}^m	1.7952				IG	0.5000	0.2000
σ_{by}^m	4.9963				IG	0.5000	0.2000

Notation of Model Parameters

	Parameters
Debt to annualized GDP ratio	s_b
Steady-state growth rate	$100 \ln \mu$
Steady state inflation rate	$100 \ln \Pi$
Inverse Frisch elasticity	ξ
Share of hand-to-mouth households	μ
Wage Calvo parameter	ω_w
Price Calvo parameter	ω_p
Capital utilization cost	ψ
Investment adjustment cost	s
Wage inflation indexing parameter	χ_w
Price inflation indexing parameter	χ_p
Habits in consumption	θ
Substitutability of private vs. gov. consumption	α_G

Notation of Model Parameters	
	Parameters
Taylor rule response to output	ϕ_y
Taylor rule response to inflation	ϕ_π
Transfers response to output	ϕ_{zy}
Inverse Frisch elasticity	ξ
Government consumption response to debt	γ_G
Tax on capital response to debt	γ_K
Tax on labor response to debt	γ_L
Transfers response to debt	γ_Z
Serial correlation on interest rate in Taylor rule	ρ_r
Serial correlation on government consumption rule	ρ_G
Serial correlation on transfers rule	ρ_Z
Serial correlation on capital tax rule	ρ_K
Serial correlation on labor tax rule	ρ_L
Serial correlation on consumption tax rule	ρ_C

Notation of Model Parameters

	Parameters
AR coefficient on government consumption policy shocks	ρ_{eG}
AR coefficient on funded transfers' shocks	ρ_{eZ}^{M}
AR coefficient on unfunded transfers' shocks	ρ_{eZ}^{F}
AR coefficient on technology shocks	ρ_{a}
AR coefficient on preference shocks	ρ_{b}
AR coefficient on monetary policy shocks	ρ_{m}
AR coefficient on investment shocks	ρ_{i}
AR coefficient on risk premium shocks	ρ_{rp}
AR coefficient on inflation drift shocks	$\rho_{\pi}NKPC$

Notation of Model Parameters

	Parameters
Standard deviation government consumption shocks	σ_G
Standard deviation funded transfers' shocks	σ_Z^M
Standard deviation unfunded transfers' shocks	σ_Z^F
Standard deviation technology shocks	σ_a
Standard deviation preference shocks	σ_b
Standard deviation monetary policy shocks	σ_m
Standard deviation investment shocks	σ_i
Standard deviation wage markup shocks	σ_w
Standard deviation price markup shocks	σ_p
Standard deviation risk premium shocks	σ_{rp}
Standard deviation inflation drift shocks	σ_{π^*}
Measurement error on GDP	σ_{GDP}^m
Measurement error on debt to GDP ratio	σ_{by}^m

Production function:

$$\hat{y}_t = \frac{y + \Omega}{y} \left[\alpha \hat{k}_t + (1 - \alpha) \hat{L}_t \right]. \tag{1}$$

Capital-labor ratio:

$$\hat{r}_t^K - \hat{w}_t = \hat{L}_t - \hat{k}_t. \tag{2}$$

Marginal cost:

$$\widehat{mc}_t = \alpha \hat{r}_t^k + (1 - \alpha) \hat{w}_t. \tag{3}$$

Phillips curve:

$$\hat{\pi}_t = \frac{\beta}{1 + \chi_p \beta} E_t \hat{\pi}_{t+1} + \frac{\chi_p}{1 + \chi_p \beta} \hat{\pi}_{t-1} + \kappa_p \widehat{mc}_t + \kappa_p \hat{\eta}_t^p, \tag{4}$$

where $\kappa_p = [(1 - \beta\omega_p)(1 - \omega_p)] / [\omega_p (1 + \beta\chi_p)]$.

Saver household's FOC for consumption:

$$\hat{\lambda}_t^S = \hat{F}_t^b - \frac{\theta}{e^{\gamma} - \theta}\hat{F}_t^a - \frac{e^{\gamma}}{e^{\gamma} - \theta}c_t^{*S} + \frac{\theta}{e^{\gamma} - \theta}c_{t-1}^{*S} - \frac{\tau^C}{1 + \tau^C}\hat{\tau}_t^C, \tag{5}$$

where $\hat{F}_t^a = u_t^a - \gamma$.
Public/private consumption in utility:

$$\hat{c}_t^* = \frac{c^S}{c^S + \alpha_G g}\hat{c}_t^S + \frac{\alpha_G g}{c^S + \alpha_G g}\hat{g}_t. \tag{6}$$

Euler equation:

$$\hat{\lambda}_t^S = \hat{R}_t + E_t\hat{\lambda}_{t+1}^S - E_t\hat{\pi}_{t+1} - E_t\hat{F}_{t+1}^a. \tag{7}$$

Maturity structure of debt:

$$\hat{R}_t + \hat{P}_t^B = \frac{\rho}{R} E_t \hat{P}_{t+1}^B . \tag{8}$$

Saver household's FOC for capacity utilization:

$$r_t^k - \frac{\tau^K}{1-\tau^K} \hat{\tau}_t^K = \frac{\psi}{1-\psi} \hat{\nu}_t . \tag{9}$$

Saver household's FOC for capital:

$$\hat{q}_t = E_t \hat{\pi}_{t+1} - \hat{R}_t + \beta e^{-\gamma} \left(1 - \tau^K\right) r^k E_t \hat{r}_{t+1}^k - \beta e^{-\gamma} \tau^K r^k E_t \hat{\tau}_{t+1}^K + \beta e^{-\gamma} (1-\delta) E_t \hat{q}_{t+1} . \tag{10}$$

Saver household's FOC for investment:

$$\hat{\imath}_t + \frac{1}{1+\beta} \hat{F}_t^a - \frac{1}{(1+\beta) s e^{2\gamma}} \hat{q}_t - \hat{F}_t^i - \frac{\beta}{1+\beta} E_t \hat{\imath}_{t+1} - \frac{\beta}{1+\beta} E_t \hat{F}_{t+1}^a = \frac{1}{1+\beta} \hat{\imath}_{t-1} . \tag{11}$$

Effective capital:

$$\hat{k}_t = \hat{\upsilon}_t + \widehat{\bar{k}}_{t-1} - \hat{F}^a_t. \tag{12}$$

Law of motion for capital:

$$\widehat{\bar{k}}_t = (1-\delta)\, e^{-\gamma} \left(\widehat{\bar{k}}_{t-1} - \hat{F}^a_t\right) + \left[1 - (1-\delta)\, e^{-\gamma}\right] \left[(1+\beta)\, se^{2\gamma} + \hat{\imath}_t\right]. \tag{13}$$

Hand-to-mouth household's budget constraint:

$$\tau^C c^N \hat{\tau}^C_t + \left(1+\tau^C\right) c^N \hat{c}^N_t = \left(1-\tau^L\right) wL \left(\hat{w}_t + \hat{L}_t\right) - \tau^L wL\hat{\tau}^L_t + z\hat{z}_t. \tag{14}$$

Aggregate households' consumption

$$c\hat{c}_t = c^S (1-\mu)\, \hat{c}^S_t + c^N \mu \hat{c}^N_t. \tag{15}$$

Wage equation:

$$\begin{aligned}\hat{w}_t &= \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} - \kappa_w\left[\hat{w}_t - \xi\hat{L}_t + \hat{\lambda}_t^S - \frac{\tau^L}{1-\tau^L}\hat{\tau}_t^L\right] \\ &+ \frac{\chi^w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\chi^w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{\chi}{1+\beta}\hat{F}_{t-1}^a - \frac{1+\beta\chi-\rho_a\beta}{1+\beta}\hat{F}_t^a + \kappa_w\hat{\eta}_t^w \quad (16)\end{aligned}$$

where $\kappa_w \equiv \left[(1-\beta\omega_w)(1-\omega_w)\right] / \left[\omega_w(1+\beta)\left(1+\frac{(1+\eta^w)\xi}{\eta^w}\right)\right]$.

Aggregate resource constraint:

$$y\hat{y}_t = c\hat{c}_t + i\hat{\imath}_t + g\hat{g}_t + \psi'(1)\,k\hat{\upsilon}_t. \quad (17)$$

Government budget constraint:

$$\begin{aligned}&\frac{b}{y}\hat{b}_t + \tau^K r^k\frac{k}{y}\left[\hat{\tau}_t^K + \hat{r}_t^k + \hat{k}_t\right] + \tau^L w\frac{L}{y}\left[\hat{\tau}_t^L + \hat{w}_t + \hat{L}_t\right] + \tau^C\frac{c}{y}\left(\hat{\tau}_t^C + \hat{c}_t\right) \\ &= \frac{1}{\beta}\frac{b}{y}\left[\hat{b}_{t-1} - \hat{\pi}_t - \hat{P}_{t-1}^B - \hat{F}_t^a\right] + \frac{b}{y}\frac{\rho}{e^\gamma}\hat{P}_t^B + \frac{g}{y}\hat{g}_t + \frac{z}{y}\hat{z}_t. \quad (18)\end{aligned}$$

Fiscal Rules

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1-\rho_G)\,\gamma_G \tilde{b}^*_{t-1} + \zeta_{g,t} \tag{19}$$

$$\hat{z}_t = \phi_{zy}\hat{y}_t + \rho_Z \hat{z}_{t-1} - (1-\rho_Z)\,\gamma_Z \tilde{b}^*_{t-1} + \zeta^M_{z,t} + \zeta^F_{z,t} \tag{20}$$

$$\hat{\tau}^L_t = \rho_L \hat{\tau}^L_{t-1} + (1-\rho_L)\,\gamma_L \tilde{b}^*_{t-1} + \zeta_{\tau_L,t} \tag{21}$$

$$\hat{\tau}^K_t = \rho_K \hat{\tau}^K_{t-1} + (1-\rho_K)\,\gamma_K \tilde{b}^*_{t-1} + \zeta_{\tau_K,t} \tag{22}$$

Monetary Rule:

$$\hat{R}_t = \max\left(-\ln R_*, \rho_r \hat{R}_{t-1} + (1-\rho_r)\left[\phi_\pi \hat{\pi}^*_t + \phi_y \hat{y}_t\right]\right) + \epsilon_{R,t} \tag{23}$$

The variables with the * superscript in equations (19) to (23) above belong to the shadow economy.

The block of equations that characterize the shadow economy consists in an additional set of equations (1) to (18), where any variable that refers to the actual economy x_t is replaced by the same variable in the shadow economy x_t^*, plus the rule for the monetary authority

$$\hat{R}_t^* = \max\left(-\ln R_*, \rho_r \hat{R}_{t-1}^* + (1-\rho_r)\left[\phi_\pi \hat{\pi}_t^* + \phi_y \hat{y}_t^*\right]\right) + \epsilon_{R,t} \tag{24}$$

and the rules for the fiscal authority,

$$\hat{g}_t^* = \rho_G \hat{g}_{t-1}^* - (1-\rho_G)\gamma_G \tilde{b}_{t-1}^* + \zeta_{g,t} \tag{25}$$

$$\hat{z}_t^* = \phi_{zy}\hat{y}_t^* + \rho_Z \hat{z}_{t-1}^* - (1-\rho_Z)\gamma_Z \tilde{b}_{t-1}^* + \zeta_{z,t}^M \tag{26}$$

$$\hat{\tau}_t^{L*} = \rho_L \hat{\tau}_{t-1}^{L*} + (1-\rho_L)\gamma_L \tilde{b}_{t-1}^* + \zeta_{\tau_L,t} \tag{27}$$

$$\hat{\tau}_t^{K*} = \rho_K \hat{\tau}_{t-1}^{K*} + (1-\rho_K)\gamma_K \tilde{b}_{t-1}^* + \zeta_{\tau_K,t}. \tag{28}$$