THE CONSTRAINT ON PUBLIC DEBT WHEN $r < g$ BUT $g < m$

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Workshop on Debt and Fiscal Policy

The debt dynamics: notation

$$\underbrace{db_t}_{\text{Total deficit}} = \underbrace{\left(\frac{s_t}{b_t}\right) b_t dt}_{\text{Primary deficit}} + \underbrace{r_t b_t dt}_{\text{Return to debtholders}}$$

Exogenous: s_t/b_t

Endogenous: $b_t = v_t B_t, \quad r_t = \xi + (1-\xi) dv_t/v_t$

Balanced growth path: $db_t/b_t \to g$

Mathematics: the integral form

- Let $\{d_t\}$ be a sequence with the property that: $\bar{d}_t = \int_0^t d_s ds$

$$\limsup_{t\to\infty} \bar{d}_t > \limsup_{t\to\infty} \bar{g}_t$$

- Then, can write integral form of the differential equation:

$$b_0 = -\int_0^\infty e^{-\bar{d}_t t} s_t dt + \int_0^\infty e^{-\bar{d}_t t}(d_t - r_t) b_t dt$$

$$\frac{b_0}{y_0} = \int_0^\infty e^{-(\bar{d}_t - \bar{g}_t)t} \left(-\frac{s_t}{y_t}\right) dt + \int_0^\infty e^{-(\bar{d}_t - \bar{g})t} \left(\frac{(d_t - r_t)b_t}{y_t}\right) dt$$

- This is a mathematical identity that holds for any convergent sequence $\{d_t\}$ satisfying that property, no economics in it

Conventional analysis with d=r

$$\frac{b_0}{y_0} = \int_0^\infty e^{-(\bar{r}_t - \bar{g}_t)t} \left(-\frac{s_t}{y_t} \right) dt \qquad \frac{Debt}{GDP} = EPV_{d-g} \left(\frac{PrimaryBalance}{GDP} \right)$$

- Debt sustainability measurement
  - Forecasting future surplus s_t: policy changes, commitments
  - Fiscal rules $s_t =$ - σb_t : hard to estimate.
  - Laffer curves and how s_t and r_t are related
- Debt sustainability tradeoff
  - Austerity wars: lower s_t may lower g_t
  - Default on b_t to lower LHS may raise r_t and lower RHS
  - Structural reforms raising g_t and r_t

With r<g this is just mathematically wrong

- Bad maths:

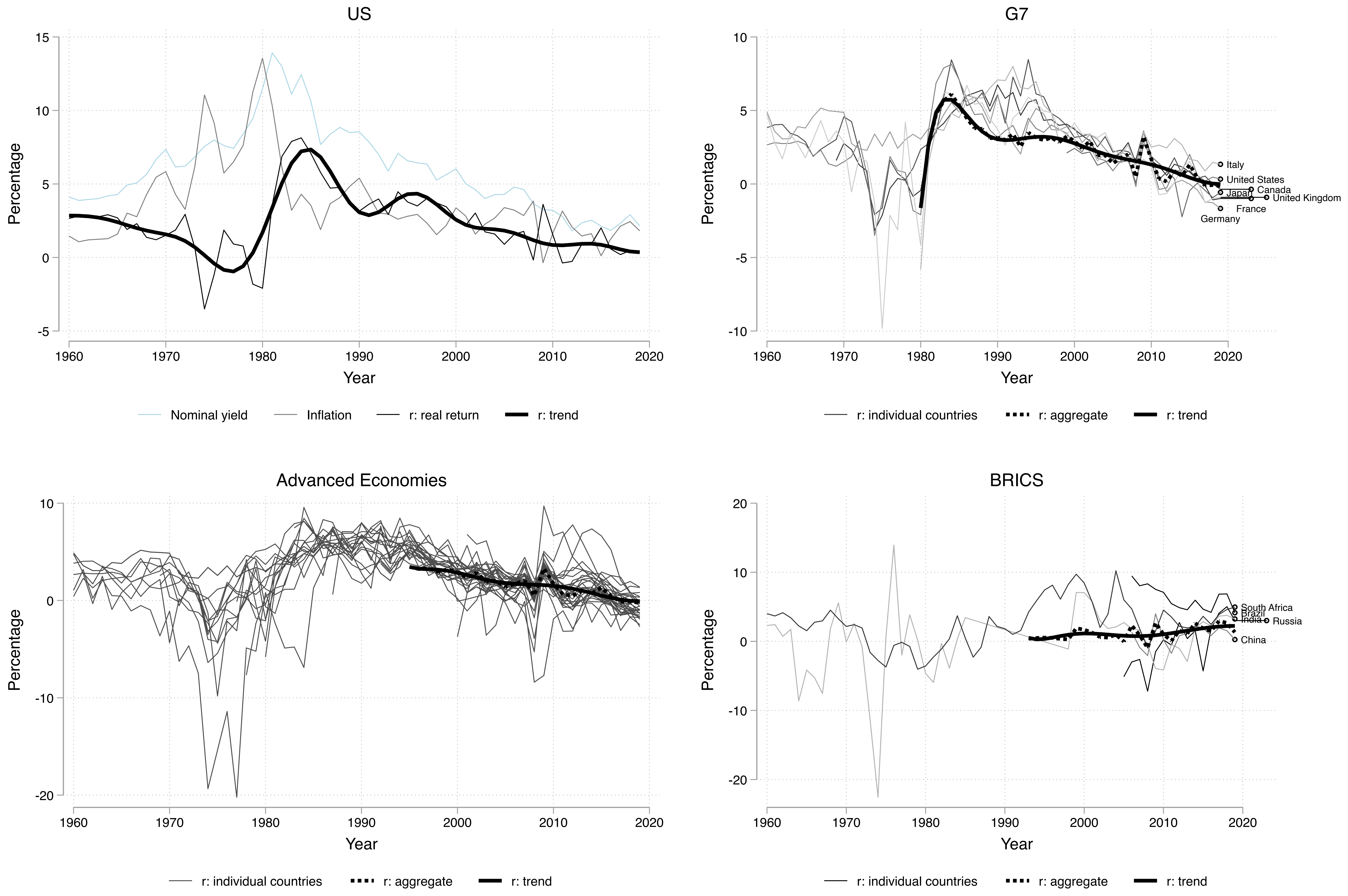
$$b_0 = \lim_{T\to\infty} \underbrace{\left[-\int_0^T e^{-\bar{r}_t t} s_t dt + e^{-\bar{r}_T T} b_T \right]}_{-\infty+\infty?}$$

- But which d_t to use? So many arbitrary choices…

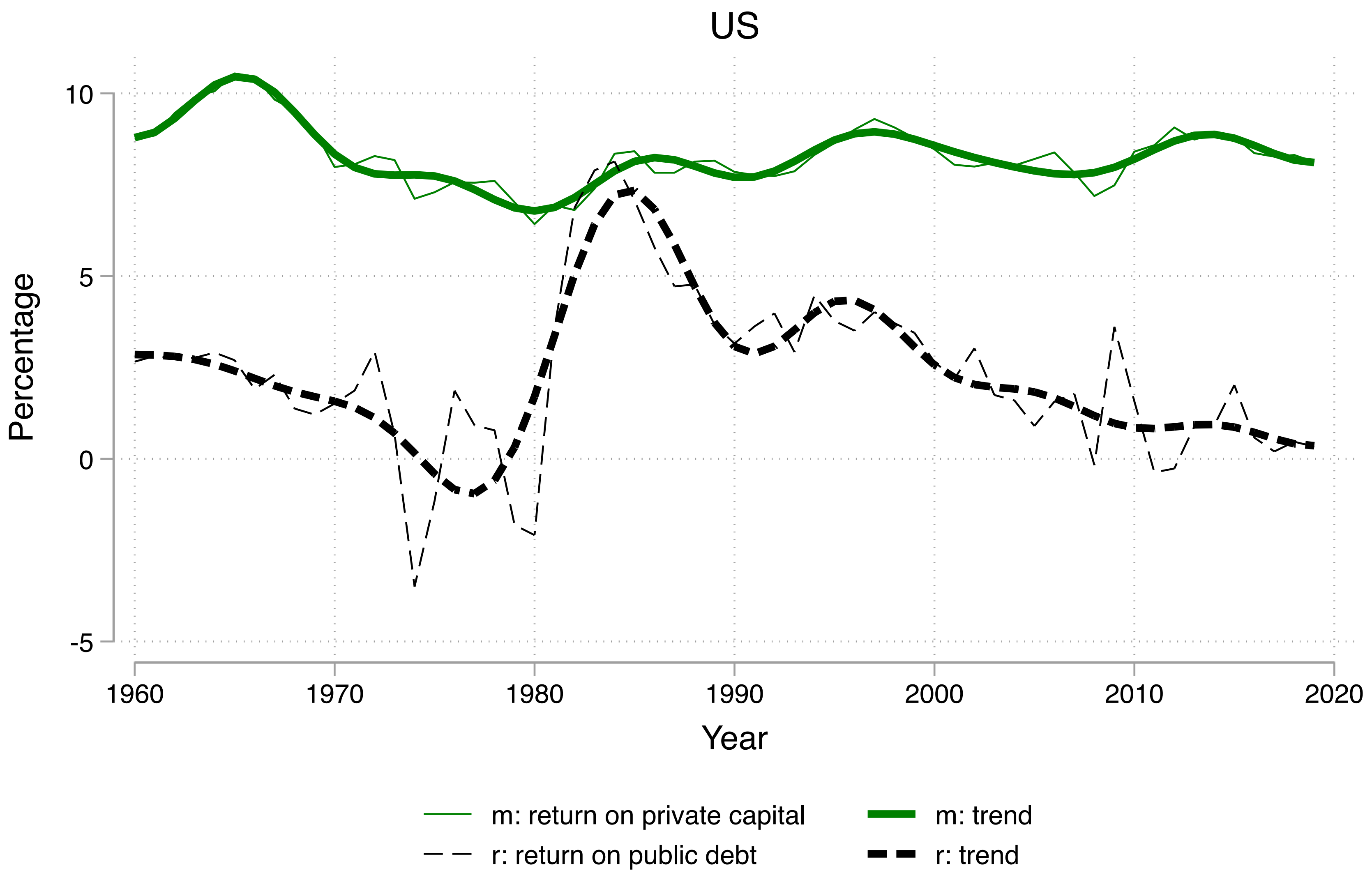
- Proposal: use returns to capital as opposed to returns to debt m_t

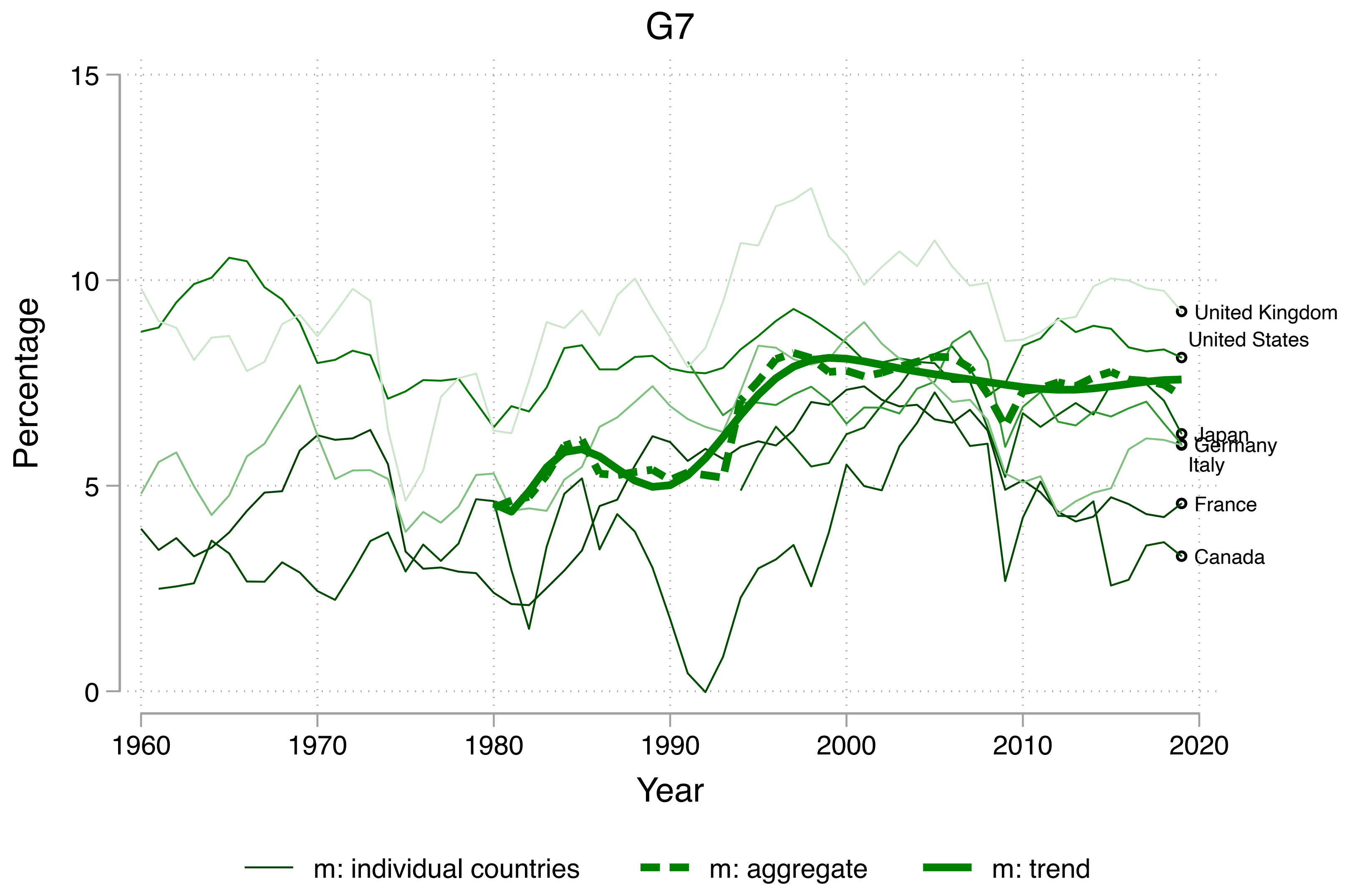
$$b_0 = -\int_0^\infty e^{-\bar{m}_t t} s_t dt + \int_0^\infty e^{-\bar{m}_t t} (m_t - r_t) b_t dt.$$

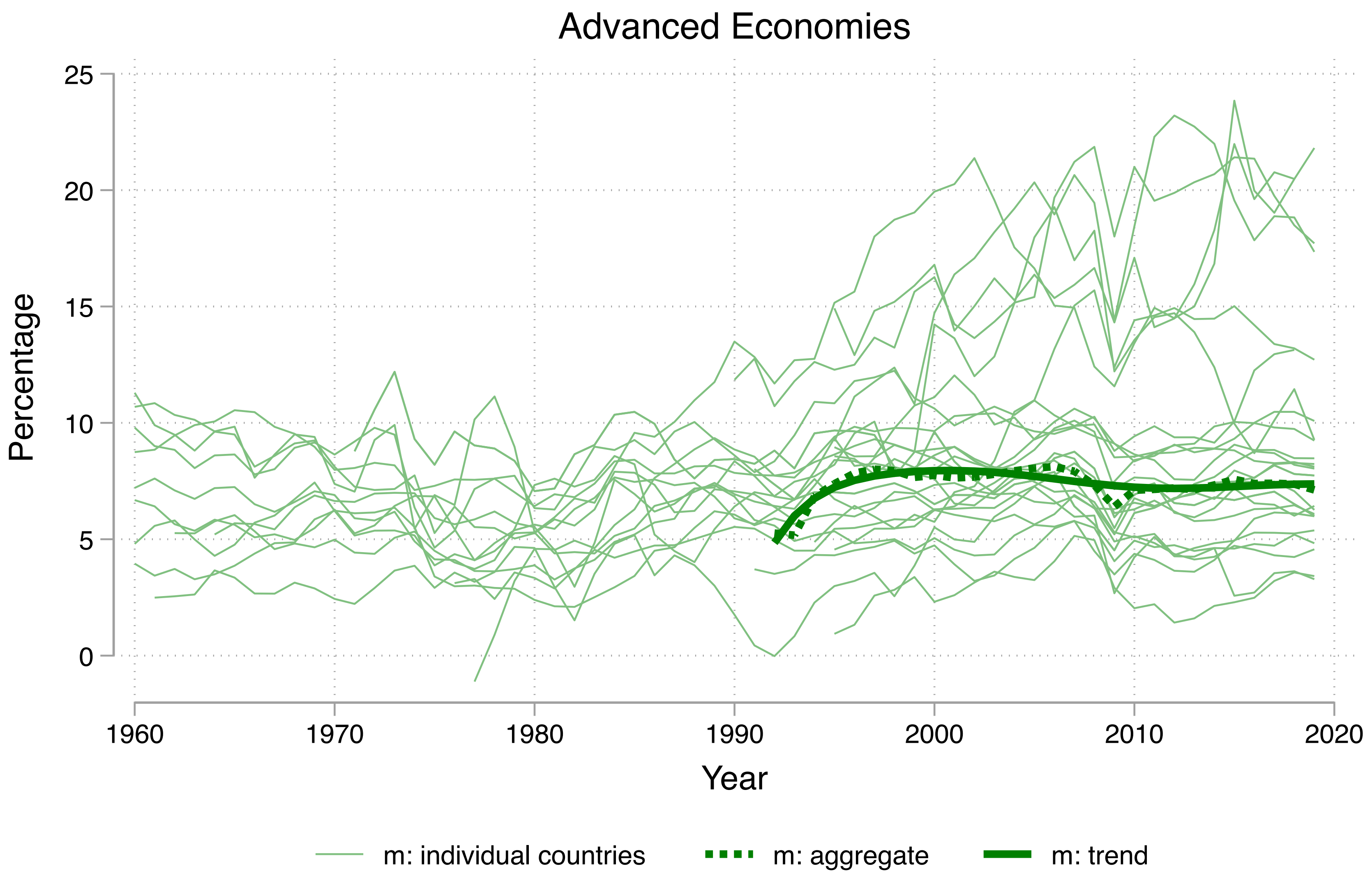
Is r<g valid? Not really these days

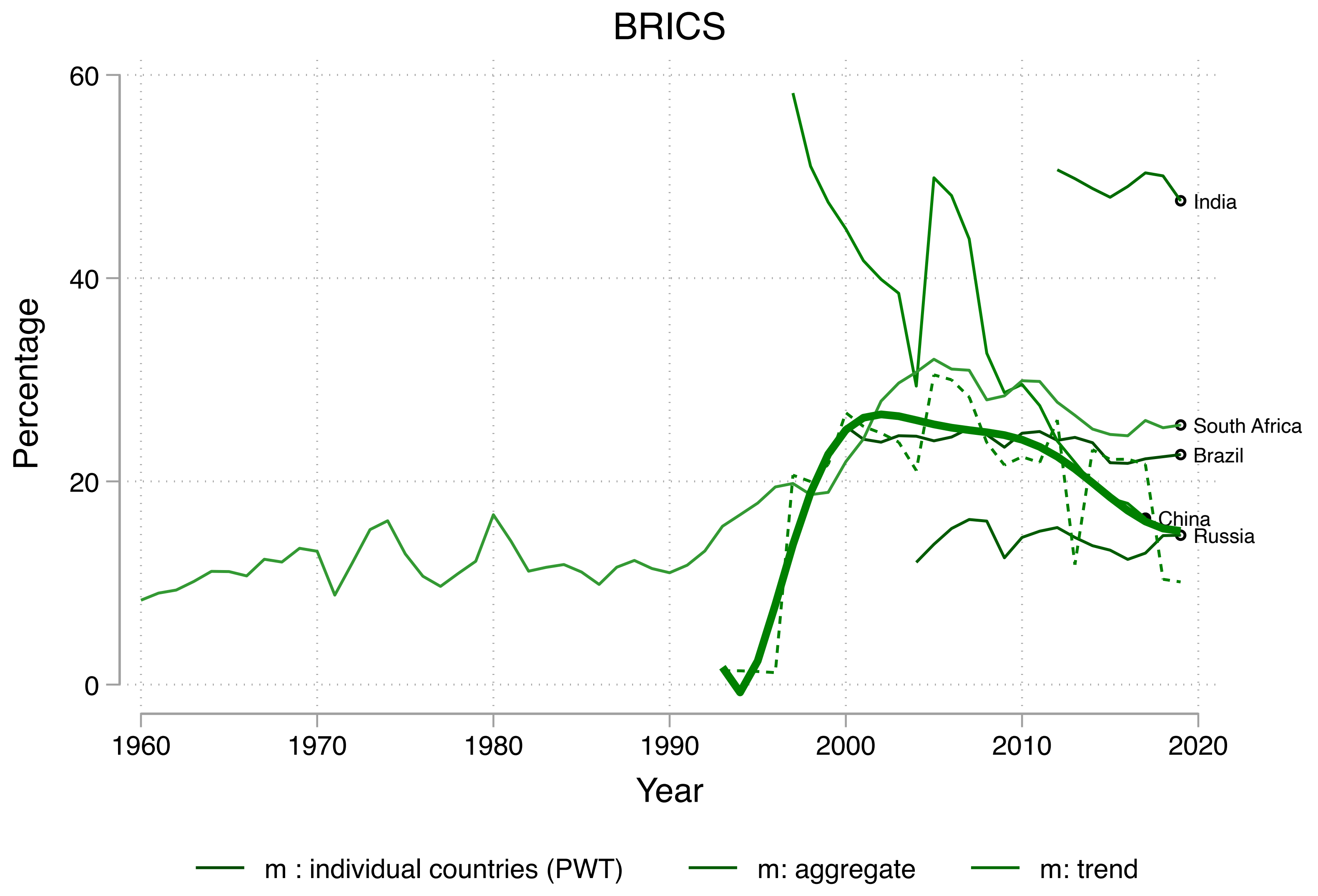


But what about d=m?

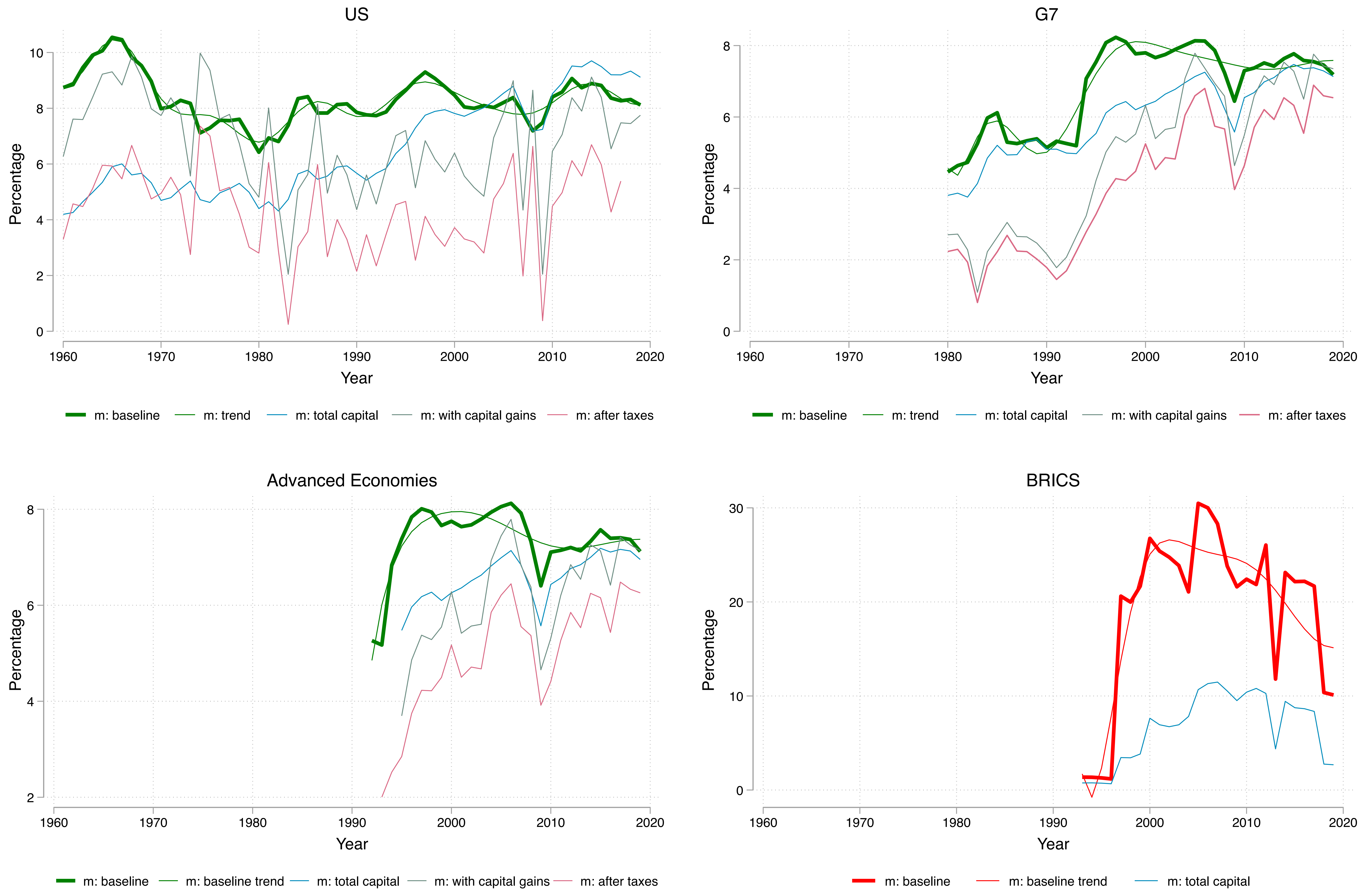




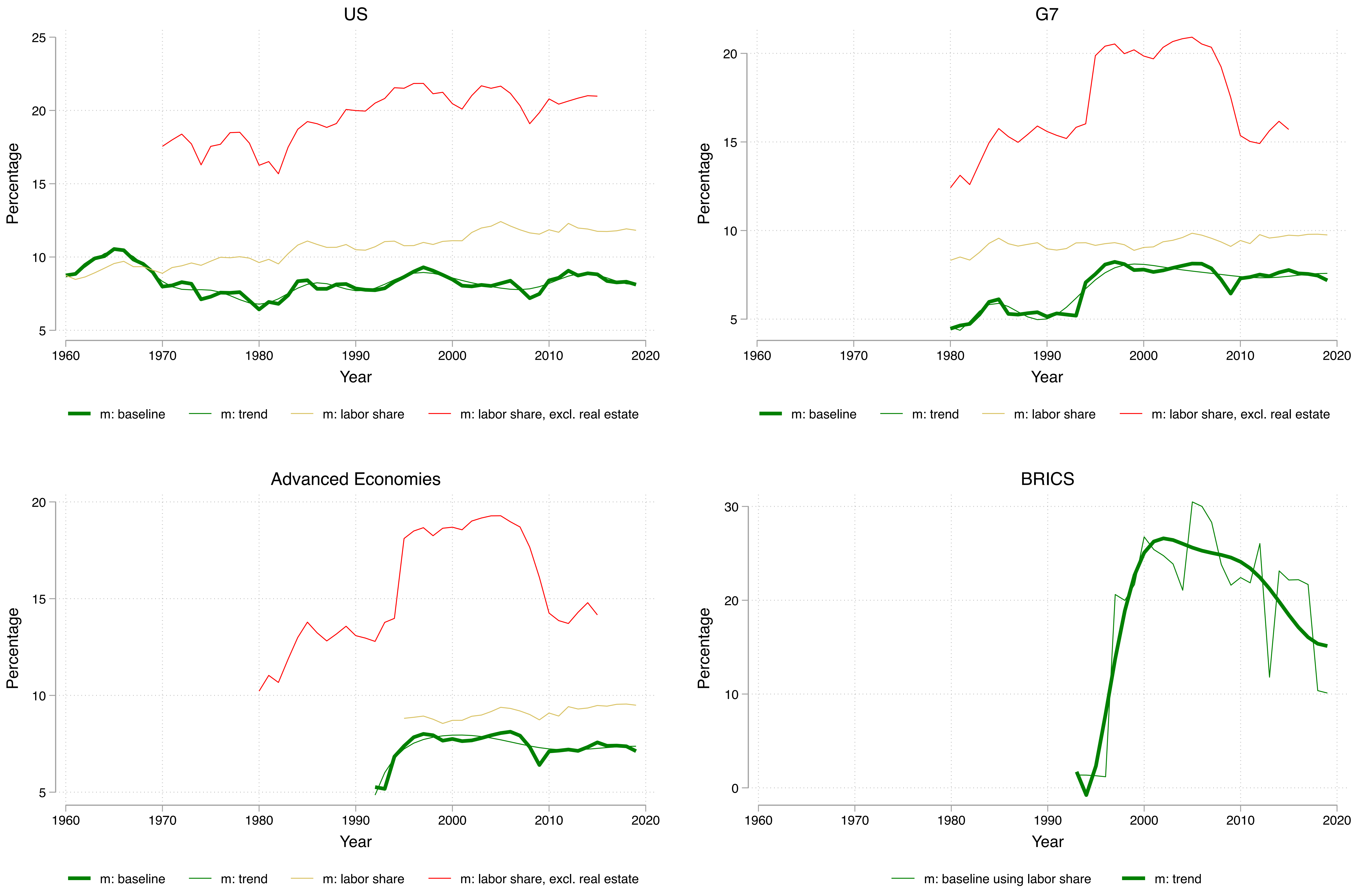




Alternatives for m: k and surpluses



Alternatives for m: labor share



The revenue from issuing debt

Debt arithmetics: PVs and debt-revenue term

$$b_0 = -\int_0^\infty e^{-\bar{m}_t t} s_t dt + \int_0^\infty e^{-\bar{m}_t t}(m_t - r_t) b_t dt.$$

- Why this makes sense: first term is the market present value of surpluses.
  - Finance: valid SDF even when markets are incomplete
  - Macro: since marginal holder of the debt can invest in capital
  - $m > g$ is a transversality condition, dynamic efficiency condition
- What this means for second term: a debt-revenue term
  - *m-r* is a bubble premium / liquidity premium / safety premium / convenience yield / seigniorage / repression.
  - *m-g* is discounting for present values

Why is second term exciting?

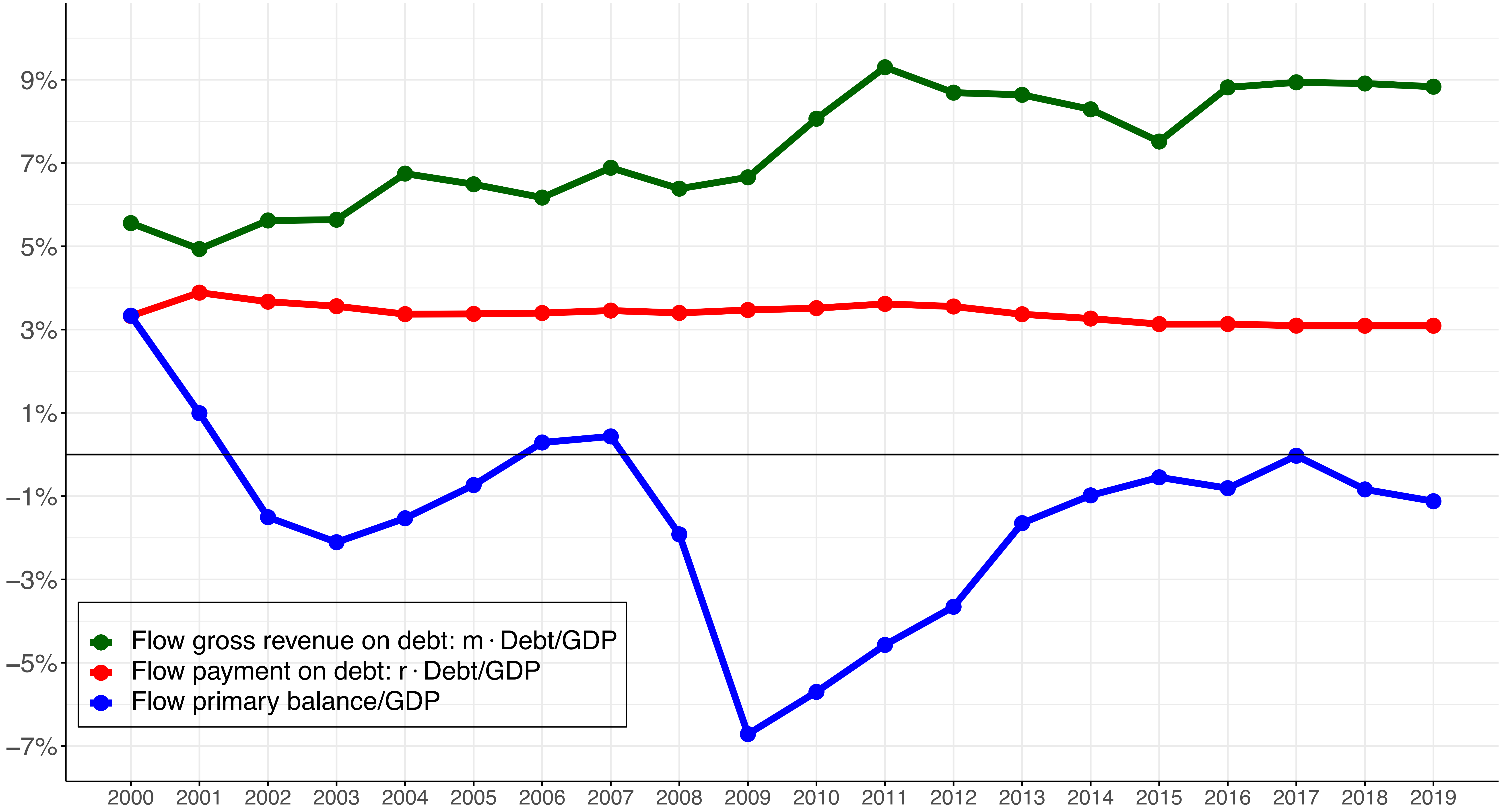
$$b_0 = -\int_0^\infty e^{-\bar{m}_t t} s_t dt + \int_0^\infty e^{-\bar{m}_t t}(m_t - r_t) b_t dt.$$

Debt/GDP = EPV$_{m\text{-}g}$(PrimaryBalance/GDP) + EPV$_{m\text{-}g}$ ((m-r)Debt/GDP)

- The revenue for the government from supplying public debt that for some reasons people are willing to hold in spite of its lower return.
- It may be very high
  - At the same time as research finds a small first term: valuation puzzles
- It comes with different policy trade-offs

Measurement: can be very large

G7 countries



m-r = 4% seems quite good, and so debt revenue of 75-100% of GDP.
But could be twice as high, or closer to zero.

A model where liquidity and safety are scarce

This paper: investigate trade-offs in theory

- Model for why is $m > g > r$?
- How does spending affect equilibrium $m - r$ and b and so the debt-revenues?
- What is the largest spending S such that debt has positive value in equilibrium ?
- How do limits on spending depend on fiscal rules ?
- How do policies affect feasible spending s/b and S ?

Rest of paper:

Offer a model that endogenizes m, r, g, S *to answer these questions.*

Show the extra "debt revenue" term has new predictions for debt sustainability

- *Reis (2013), Aoki, Benigno, Kiyotaki (2010), Farhi, Tirole (2012), Aoki, Nakajima, Nikolov (2014), Hirano, Inaba, Yanagawa (2015), Di Tella (2020), Bassetto Cui (2018), Kocherlakota (2021), Acharya Rajan (2021)*
- *Barro (2020), Mehrotra Sergeyev (2020). Kocherlakota (2021), Ball Mankiw (2021), Mian, Sufi, Straub (2021), Liu, Schmid, Yaron (2021)*
- *Jiang, Lustig, van Niewerburgh, Xialoan (2020, 2021), Brunnermeier, Merkel, Sannikov (2020),*

Production side

Capital heterogeneity:

- Ex ante capital types q

$$q \in [0,1], \quad Q(q)$$

- Ex post depreciation risk

$$\delta(q) dz_t^{qi}$$

- Simpler case, no risk, 2 types

$$q_t = \begin{cases} 1 & \text{if type } H, \ \text{share } \alpha \\ 0 & \text{if type } L, \ \text{share } 1-\alpha \end{cases}$$

Neoclassical firm

- Maximizes (with free entry)

$$\max \left\{ \int \int \left[m_t q_t dt - r_t^{qi} dt - \delta dz_t^{qi} \right] k_t^{qi} dQ(q) di \right\}$$

- Pays marginal return

$$r_t^{qi} dt = m_t q_t dt - \delta(q) dz_t^{qi}$$

- If only better type used, ***q=1, δ(q)=0***

$$\max \left\{ \left[m_t - r_t \right] k_t dt \right\}$$

Consumers and financial markets

Households

$$\max_{\{c_t^{qi}, b_t^{qi}, l_t^{qi}, k_t^{qi}\}} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log c_t^{qi} dt \right]$$

$$\text{subject to: } a_t^{qi} = b_t^{qi} + l_t^{qi} + k_t^{qi} \quad \text{with} \quad b_t^{qi} \geq 0, k_t^{qi} \geq 0$$

$$da_t^{qi} = (r_t b_t^{qi} + r_t^l l_t^{qi} + r_t^{qi} k_t^{qi} - c_t^{qi}) dt$$

$$-r_t^l l_t^{qi} \leq \gamma m_t q_t k_t^{qi}$$

Markets

$$\int \int l_t^{qi} dQ(q) di = 0 \quad \text{and} \quad \int \int b_t^{qi} dQ(q) di = b_t.$$

Steady state govt budget constraint

$$b = \left\{ \frac{s}{g - r} \quad , \quad 0 \right\}$$

Eqm: (r, $\kappa = b/k$, g) given an exogenous $\sigma = s/b$. Restrict to $r < g < m$ and $\sigma \geq 0$.

Why m > r > g in the first place?

- If **no credit frictions** $\gamma = 1$: all lend to type $q = 1$, AK economy, $r = m > g$
- Misallocation: some households/sectors have **inferior investment opportunities**.
  - Lending limited by inability to ensure will be paid back
  - **Public debt** provides alternative to save, store or value, liquidity
  - **Gap** $m - r$ higher when underdeveloped financial markets
- Investment risk: the **capital** has a risky idiosyncratic return
  - Financial markets are incomplete, investment requires bearing risk
  - **Public debt** is aggregate savings vehicle, has a safe return
  - **Gap** $m - r$ driven by risk and search for refuge from it

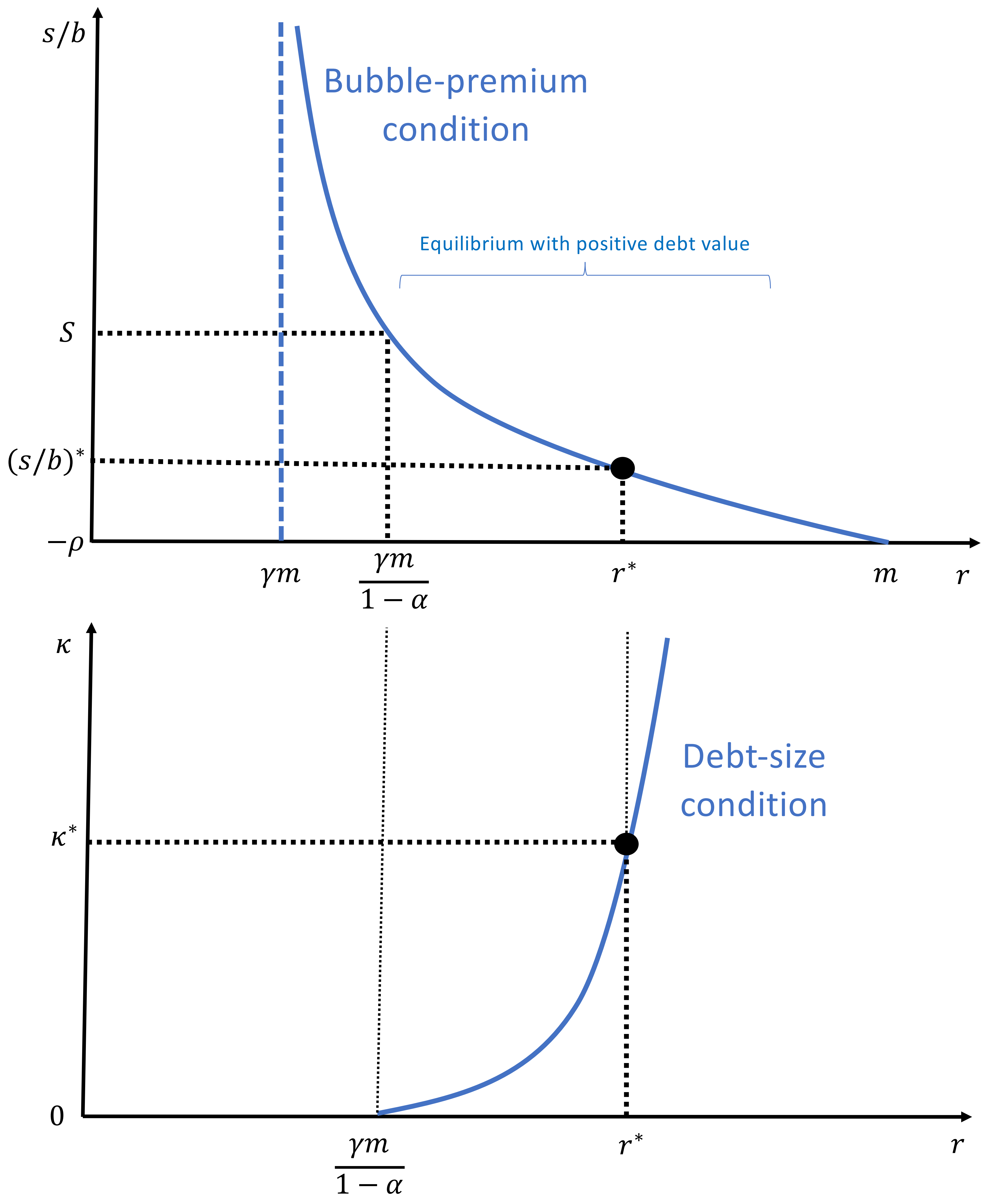
Misallocation-only model, in the BGP

- Bubble-premium equilibrium condition

$$\frac{\alpha(m-r)}{1-\frac{\gamma m}{r}} = \rho + \frac{s}{b}$$

- Debt-size equilibrium condition

$$\kappa = \frac{1-\alpha}{\alpha} - \frac{\gamma m}{\alpha r}$$



Spending and its constraints in the BGP

Proposition 1. *In the simple 2-type economy there is an equilibrium where government can run a permanent deficit paid for by the bubble premium and:*

- *More spending* (s/b) *requires a higher bubble premium* $m - r$.
- *More spending* (s/b) *lowers the ratio of public debt to private assets* κ.
- *More spending* (s/b) *increases inequality of consumption and asset growth.*
- *The fiscal capacity is:*

$$S = m\left(1 - \frac{\gamma}{1-\alpha}\right) - \rho, \tag{16}$$

so it is smaller if the marginal product of capital is lower (low m*), the economy is more financially developed (high* γ*), or if there are more high productivity types (high* α*)*

General model: investment

0 — *r/m* — *q** — *1*

Lend, save bonds	Some capital, some bonds	No bonds
l+b	*l+b+k*	*l+k*

$$\rho + \frac{s}{b} = \int_{r/m}^{q^*} \left(\frac{mq - r}{\delta(q)}\right)^2 dQ(q) + \int_{q^*}^{1} \left(\frac{mq - r}{1 - \frac{\gamma m q}{r}}\right) dQ(q)$$

$$\frac{1}{1+\kappa} = \int_{r/m}^{q^*} \left(\frac{mq - r}{\delta(q)^2}\right) dQ(q) + \int_{q^*}^{1} \frac{r}{r - \gamma m q} dQ(q)$$

Spending and its constraints in the BGP

Proposition 2. *In the general economy there is an equilibrium where government can run a permanent deficit paid for by the bubble premium and:*

- *More spending* (s/b) *requires a higher bubble premium* $m - r$.
- *More spending* (s/b) *lowers the ratio of public debt to private capital* κ.
- *More spending* (s/b) *increases inequality of consumption and asset growth between those at the top of the income distribution (with* $q > q^*$*) and those at the bottom (with* $q < r/m$*).*
- *There is a finite fiscal capacity* S*, which is smaller if the marginal product of capital is lower (low* m*), the economy is more financially developed (high* γ*), if there are more high productivity types (lower* $G(q^*)$*), or if there is less idiosyncratic risk in the economy (weakly lower* $\delta(q)$ *for all* q*)*

Fiscal rules and transitions

No transitions

- Rule was:

$$s_t = \sigma b_t$$

- Initial condition

$$B_0$$

- Since

$$b_0 = v_0 B_0$$

price v_0 jumps, economy goes to BGP right away.

- If $\sigma > S$, then $v_0 = 0$, and BGP is $b = s = 0$.

Alternative: fiscal rule and one-period debt

Now $s/a = \sigma_a$ is exogenous

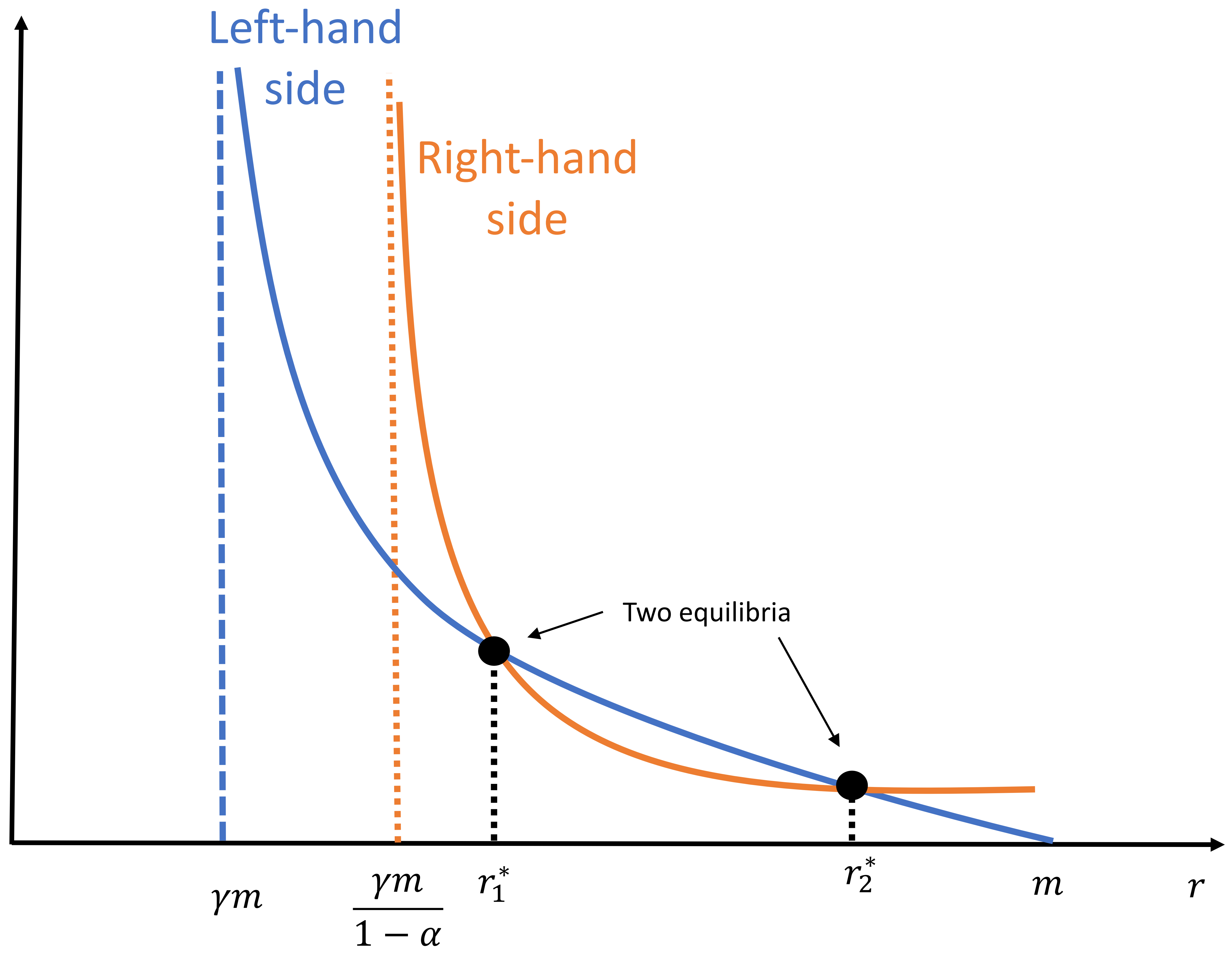
$$\frac{\alpha(m-r)}{1-\frac{\gamma m}{r}} = \rho + \frac{s}{a}\left(1 - \frac{\alpha}{1-\gamma m/r}\right)^{-1}$$

BGP:

- Still fiscal capacity, like proposition 1
- But can have high-r and high-b case.

Transition dynamics:

- if $b_0 > b'$, converge to b^*
- If $\sigma_a > S$ debt explodes, then $r > g$



Diminishing returns and transitions

Now production function $y_t = A_t K_t^\theta$

- If TFP grows at *$g/(1\text{-}\theta)$*, now exogenous. But MPK is now endogenous

$$m_t = \theta \left(\frac{Y_t}{K_t} \right)$$

- In steady state, same equilibrium conditions. Now, more spending lowers *r*, substitute to *K*, lowers *m*. Higher *S*

- In transition bubble premium may rise or fall depending on fiscal rule

Extension: exorbitant privilege

Now extra foreign demand function for bonds B(r-r^f)

- Debt-size condition now becomes:

$$\frac{1}{1+\kappa} + \frac{B(r)}{a_0} = \int_{r/m}^{q^*} \left(\frac{mq - r}{\delta(q)^2} \right) dQ(q) + \int_{q^*}^{1} \left(\frac{r}{r - \gamma mq} \right) dQ(q).$$

- If perfect capital mobility: fixes *r*, unique ***s/b=S***
- If completely inelastic, model is unchanged, mechanically higher ***S***
- In between the two, proposition holds just the same. But lower interest rates now lower bond-holdings by more, reach ***S*** faster.

Policy questions

Monetary-fiscal policy trade-offs

Two questions:

1) Does the policy change lower/raise the fiscal capacity S ?

2) Does the policy change, keeping r fixed, lower/raise s/b ? Call it fiscal space.

Financial repression

If have coerced debt

$$b_t = b_t^c + b_t^v$$

$$db_t = s_t dt + r_t b_t^v dt$$

Proposition: An increase in the share of public debt that is coerced raises the fiscal capacity and the fiscal space.

Financial repression imposes a repression premium on the coerced debt.

Unambiguously loosens government debt constraint, quick solution to troubles.

But lowers growth, worsens allocation of capital, may trigger financial crises.

Monetary policy and inflation

If have inflation

$$dp_t/p_t = \pi_t dt + \sigma_\pi dz_t^\pi$$

$$db_t = s_t dt + r_t b_t dt - b_t \sigma_\pi dz_t^\pi$$

Proposition: Changes in expected inflation have no consequences on the spending ability of the government. A higher variance of inflation lowers the fiscal capacity and the fiscal space.

Ex post benefit, inflate the debt. But ex ante, creates an inflation risk premium, lowers debt safety premium, tightens government budget constraint

No conflict, price stability maximizes fiscal resources, prevents debt crisis

Redistributive policy

Tax high-quality, transfer to low quality

$$\rho + \frac{s}{b} = \int_{r/m}^{q^*} \left(\frac{mq - r}{\delta(q)} \right)^2 dQ(q) + (1 - \psi) \int_{q^*}^{1} \left(\frac{mq - r}{1 - \frac{\gamma m q}{r}} \right) dQ(q)$$

Proposition: A larger redistribution program lowers the fiscal capacity and the fiscal space.

Because financial friction prevents flow of assets.

<u>Implication:</u> there is a conflict between a policymaker that wants to redistribute and another that wants public spending. With polarization may come more total spending, yet the bound is lower, so higher risk of bubble popping

Proportional income taxes

All income is taxed proportionately

$$da_t^{qi} = (1-\tau)(r_t b_t^{qi} + r_t^l l_t^{qi} + r_t^{qi} k_t^{qi})dt + (T_t^q - c_t^{qi})dt - \delta(q) dz_t^{qi},$$

$$-r_t^l l_t^{qi} \leq (1-\tau)\gamma m_t q_t k_t^{qi}.$$

Proposition: A marginal increase in the tax rate on all income, that is rebated to each type of household, raises fiscal capacity, but lowers fiscal space.

Direct effect raise revenues, indirect effect lowers borrowing from productive types, increases misallocation of capital

Implication: to measure if tax cuts pay for themselves, measure effect on capital allocation, through impact on ***m* - *r***

Conclusion

Conclusion

What is the constraint on public debt when there is a bubble ($r<g$) but the economy is dynamically efficient ($m>g$)?

Three points:

- r has fallen, but m has not. Careful with r-m models/predictions.
- There is a debt revenue term, and it can be substantial
- One model of the debt revenue term driven by misallocation of capital, competition between public debt and private credit markets
- Very different take on monetary/fiscal policies regarding debt sustainability

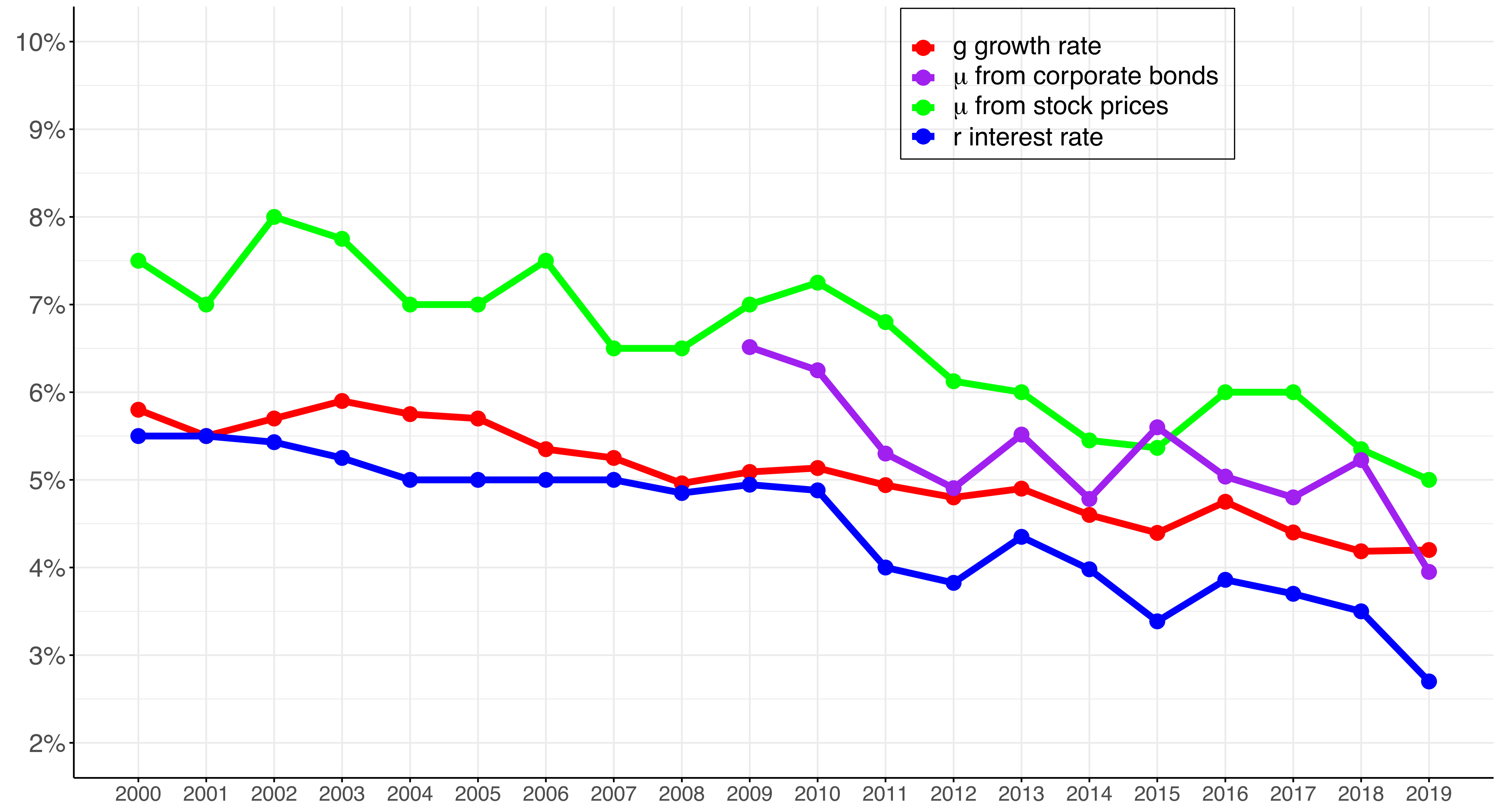
Extras

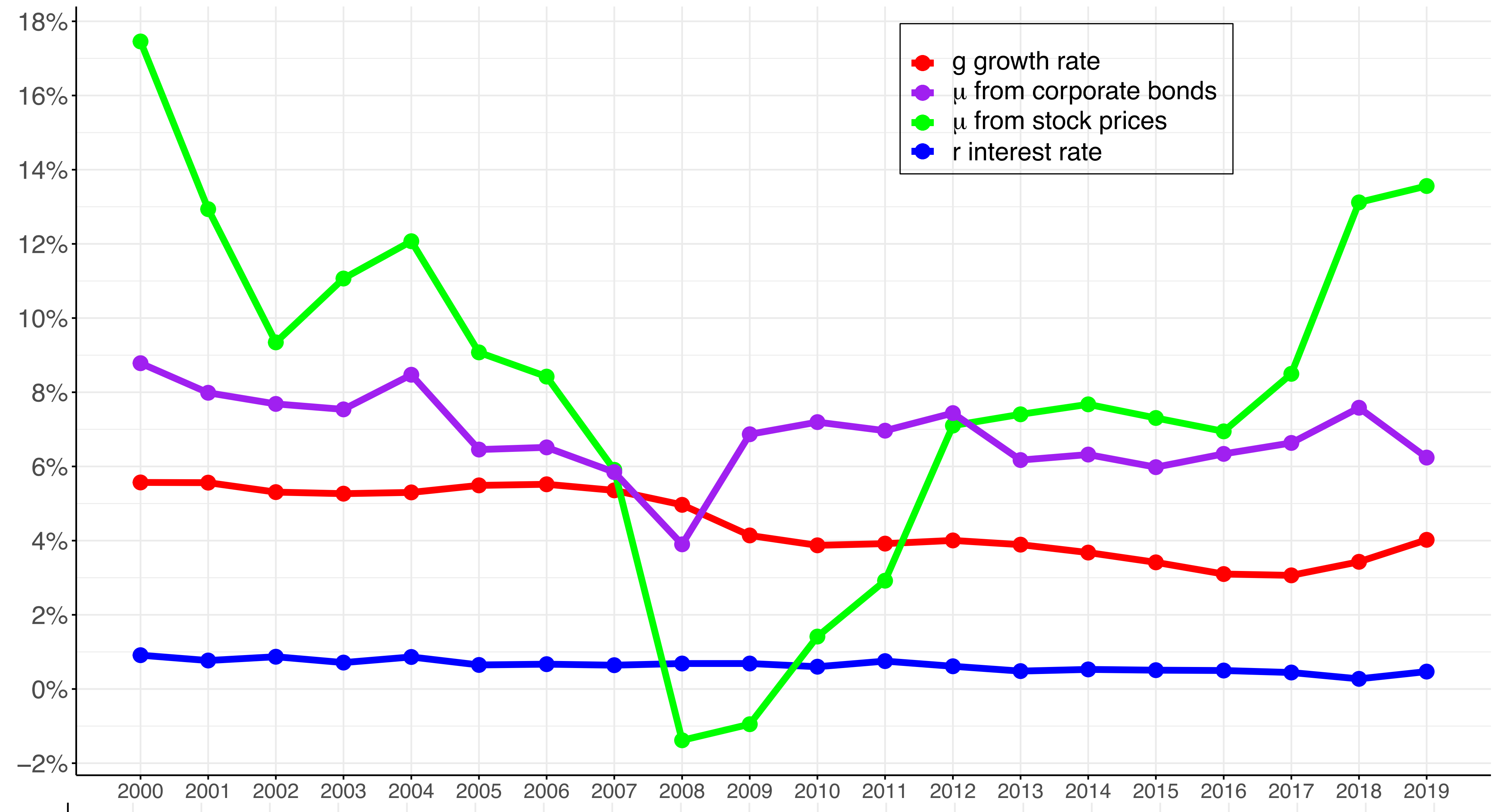
With uncertainty

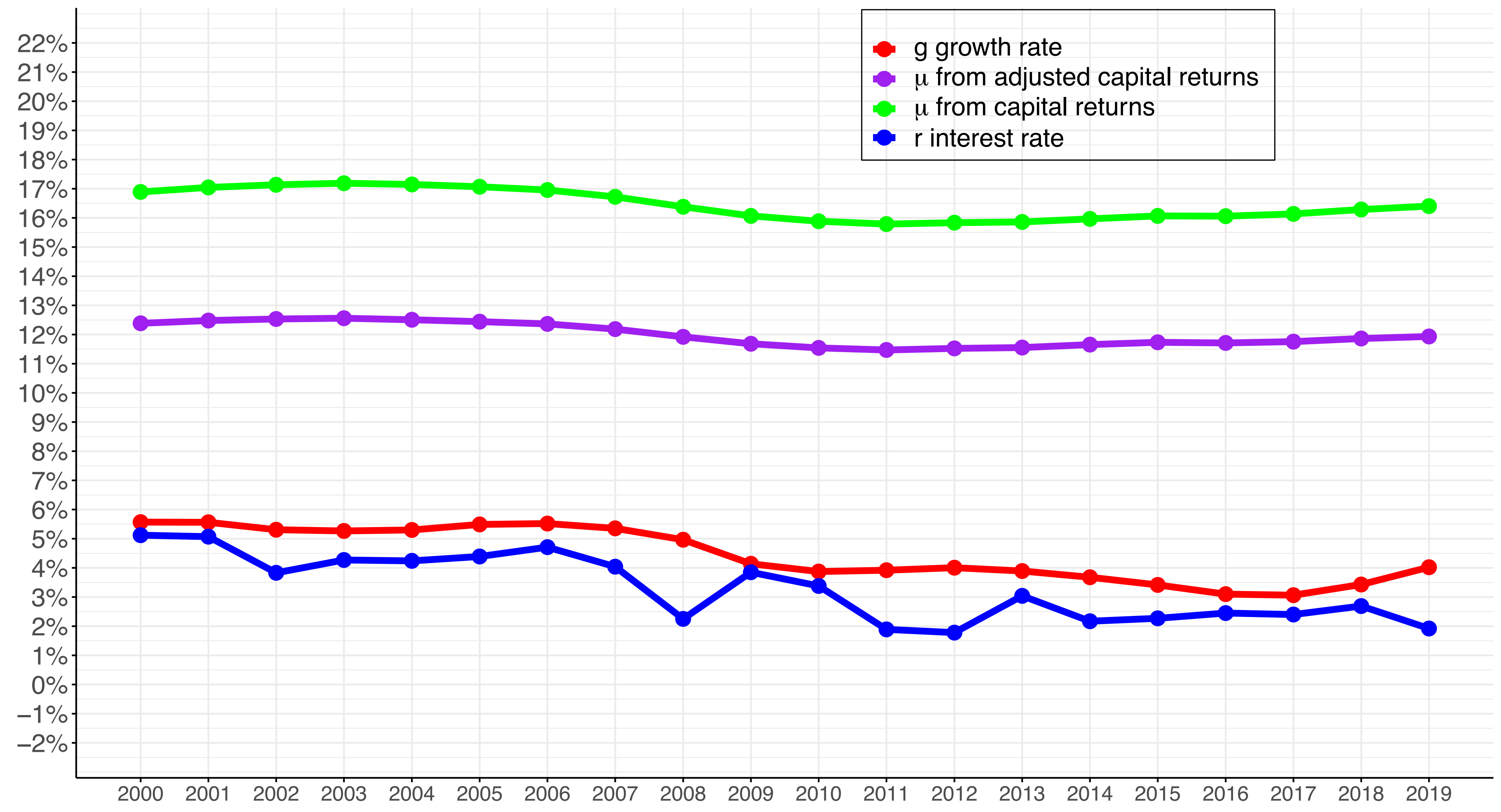
$$\lim_{t\to\infty} \mathbb{E}_t \left[(D_t/D_0) e^{\bar{g}_t t} (b_t/y_t) \right] = 0$$

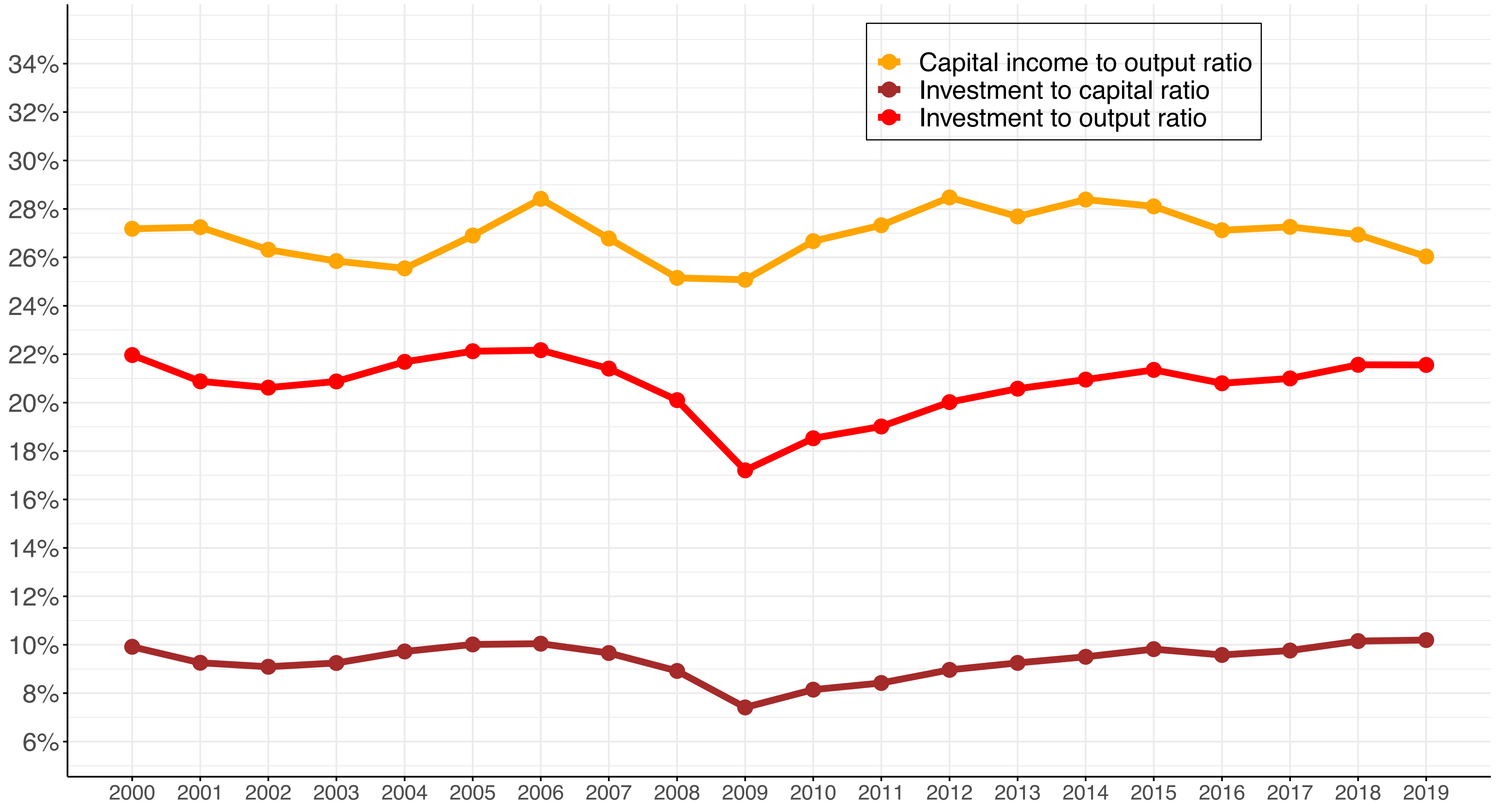
$$b_0 = -\,\mathbb{E}_t \left(\int_0^\infty \frac{D_t s_t}{D_0} dt \right) + \mathbb{E}_t \left(\int_0^\infty \frac{D_t (m_t - r_t) b_t}{D_0} dt \right),$$

US data since 2000: r < g < m









Application: aggregate uncertainty

Now depreciation risks have common component

$$dz_t^{qi} = \zeta dz_t + d\hat{z}_t^{qi}$$

- Bubble-premium conditions becomes:

$$\rho + \frac{s}{b} = \int_{r/m}^{q^*} \left(\frac{mq - r}{\delta(q)(1+\zeta)} \right)^2 dQ(q) + \int_{q^*}^{1} \frac{(mq - r}{1 - \frac{\gamma mq}{r}} dQ(q)$$

$$- \left[\int_{r/m}^{q^*} \left(\frac{mq - r}{\delta(q)(1+\zeta)} \right) dQ(q) + \int_{q^*}^{1} \frac{\delta(q)(mq - r}{1 - \frac{\gamma mq}{r}} dQ(q) \right] \zeta dz_t$$

- Bad shock lowers spending
- Change in average spending has same effects as in proposition 2
- Increase in uncertainty lowers capital holdings, raises safety premium, lower *r*

Policy lessons

Lesson 1: *One-time deficit gambles are feasible if the spending splurge keeps $r < g$*

Lesson 2: *The sign of r-g is the sign of the maximum permanent primary surplus.*

Lesson 3: *The present value of spending must stay below the bubble component of debt minus the current value of the debt.*

Lesson 4: *Higher public spending requires a higher bubble premium but comes with lower bondholdings. There is a finite fiscal capacity that shrinks with less inequality and with more financial development*

Lesson 5: *Fiscal rule followed may lead to debt runs, aggregate uncertainty in economy increases fiscal capacity, foreign demand for bonds makes it arrive sooner.*

How will the public debt be paid for?

(i) Inflation volatility (aggregate risk on debt)

- *Inflating the debt makes constraint tighter*

(ii) Financial repression (coerced holding of government bonds)

- *Makes constraint looser, but at large cost in growth and welfare*

(iii) Redistribution that taxes productive types, transfers to others

- *Successful (insurance-providing) transfers makes constraint tighter*

(iv) Rise in proportional income tax rate

- *Beyond Laffer, impact on re-allocation of capital in economy (financial markets)*

Answers to questions

- Why is $m > g > r$?

  Because of the safety and liquidity that debt offers, creating a bubble premium / convenience yield / seignorage that can pay for some spending.

- How does spending affect equilibrium bubble premium ?

  More spending raises m-r but lowers b/k, increases inequality.

- What is the largest spending S such that debt has positive value in equilibrium?

  There is a finite limit to spending. It is smaller if the economy is less productive, has more developed private financial markets, and less idiosyncratic risk

- How do policies affect feasible spending s/b and S ?

  Inflation backfires, repression works but is costly, redistribution and taxation have surprising implications and create conflicts.